7
Local Loads

Stresses caused by external local loads are a major concern to designers of pressure vessels. The techniques for analyzing local stresses and the methods of handling these loadings to keep these stresses within prescribed limits has been the focus of much research. Various theories and techniques have been proposed and investigated by experimental testing to verify the accuracy of the solutions.

Clearly the most significant findings and solutions are those developed by Professor P. P. Bijlaard of Cornell University in the 1950s. These investigations were sponsored by the Pressure Vessel Research Committee of the Welding Research Council. His findings have formed the basis of Welding Research Council Bulletin #107, an internationally accepted method for analyzing stresses due to local loads in cylindrical and spherical shells. The "Bijlaard Curves," illustrated in several sections of this chapter, provide a convenient and accurate method of analysis.

Other methods are also available for analyzing stresses due to local loads, and several have been included herein. It should be noted that the methods utilized in WRC Bulletin #107 have not been included here in their entirety. The technique has been simplified for ease of application. For more rigorous applications, the reader is referred to this excellent source.

Since this book applies to thin-walled vessels only, the detail included in WRC Bulletin #107 is not warranted. No distinction has been made between the inside and outside surfaces of the vessel at local attachments. For vessels in the thick-wall category, these criteria would be inadequate.

Other methods that are used for analyzing local loads are as follows. The designer should be familiar with these methods and when they should be applied.

1. Roark Technical Note #806.
2. Ring analysis as outlined in Procedure 7-1.
3. Beam on elastic foundation methods where the elastic foundation is the vessel shell.
4. Bijlaard analysis as outlined in Procedures 7-4 and 7-5.
5. WRC Bulletin #107.
6. Finite element analysis.

These methods provide results with a varying degree of accuracy. Obviously some are considered "ball park" techniques while others are extremely accurate. The use of one method over another will be determined by how critical the loading is and how critical the vessel is. Obviously it would be uneconomical and impractical to apply finite element analysis on platform support clips. It would, however, be considered prudent to do so on the vessel lug supports of a high-pressure reactor. Finite element analysis is beyond the scope of this book.

Another basis for determining what method to use depends on whether the local load is "isolated" from other local loads and what "fix" will be applied for overstressed conditions. For many loadings in one plane the ring-type analysis has certain advantages. This technique takes into account the additive overlapping effects of each load on the other. It also has the ability to superimpose different types of loading on the same ring section. It also provides an ideal solution for design of a circumferential ring stiffener to take these loads.

If reinforcing pads are used to beef up the shell locally, then the Bijlaard and WRC #107 techniques provide ideal solutions. These methods do not take into account closely spaced loads and their influence on one another. It assumes the local loading is isolated. This technique also provides a fast and accurate method of distinguishing between membrane and bending stresses for combining with other principal stresses.

For local loads where a partial ring stiffener is to be used to reduce local stresses, the beam on elastic foundation method provides an ideal method for sizing the partial rings or stiffener plates. The stresses in the shell must then be analyzed by another local load procedure. Shell stresses can be checked by the beam-on-elastic-foundation method for continuous radial loads about the entire circumference of a vessel shell or ring.

Procedure 7-3 has been included as a technique for converting various shapes of attachments to those which can more readily be utilized in these design procedures. Both the shape of an attachment and whether it is of solid or hollow cross section will have a distinct effect on the distribution of stresses, location of maximum stresses, and stress concentrations.

There are various methods for reducing stresses at local loadings. As shown in the foregoing paragraphs, these will have some bearing on how the loads are analyzed or how stiffening rings or reinforcing plates are sized. The following methods apply to reducing shell stresses locally.

1. Increase the size of the attachment.
2. Increase the number of attachments.
3. Change the shape of the attachment to further distribute stresses.
4. Add reinforcing pads. Reinforcing pads should not be thinner than 0.75 times nor thicker than 1.5 times the

thickness of the shell to which they are attached. They should not exceed 1.5 times the length of the attachment and should be continuously welded. Shell stresses must be investigated at the edge of the attachment to the pad as well as at the edge of the pad.

5. Increase shell thickness locally or as an entire shell course.
6. Add partial ring stiffeners.
7. Add full ring stiffeners.

The local stresses as outlined herein do not apply to local stresses due to any condition of internal restraint such as thermal or discontinuity stresses. Local stresses as defined by this section are due to external mechanical loads. The mechanical loading may be the external loads caused by the thermal growth of the attached piping, but this is not a thermal stress! For an outline of external local loads, see "Categories of Loadings" in Chapter 1.

Procedure 7-1: Stresses in Circular Rings [1–6]

Notation

R_m = mean radius of shell, in.
R_1 = distance to centroid of ring-shell, in.
M = internal moment in shell, in.-lb
M_c = external circumferential moment, in.-lb
M_h = external longitudinal moment (at clip or attachment only), in.-lb
M_L = general longitudinal moment on vessel, in.-lb
F_T = tangential load, lb
F_1,F_2 = loads on attachment, lb
f_a,f_b = equivalent radial load on 1-in. length of shell, lb
f_1 = resultant radial load, lb
P_r = radial load, lb
P = internal pressure, psi
P_e = external pressure, psi
T = internal tension/compression force, lb
K_m,K_T,K_r = internal moment coefficients
C_m,C_T,C_r = internal tension/compression coefficients
$S_{1\text{-}8}$ = shell stresses, psi
Z = section modulus, in.^{3}
t = shell thickness, in.
σ_x = longitudinal stress, psi
σ_ϕ = circumferential stress, psi
e = length of shell which acts with attachment, in.
θ = angular distance between loads or from point of consideration, degrees
W = total weight of vessel above plane under consideration, lb
A = ASME external pressure factor
A_s = metal cross-sectional area of shell, in.^{2}
A_r = cross-sectional area of ring, in.^{2}
B = allowable longitudinal compression stress, psi
E = joint efficiency
E_1 = modulus of elasticity, psi
p = allowable circumferential buckling stress, lb/in.
I = moment of inertia, in.^{4}
S = code allowable stress, tension, psi

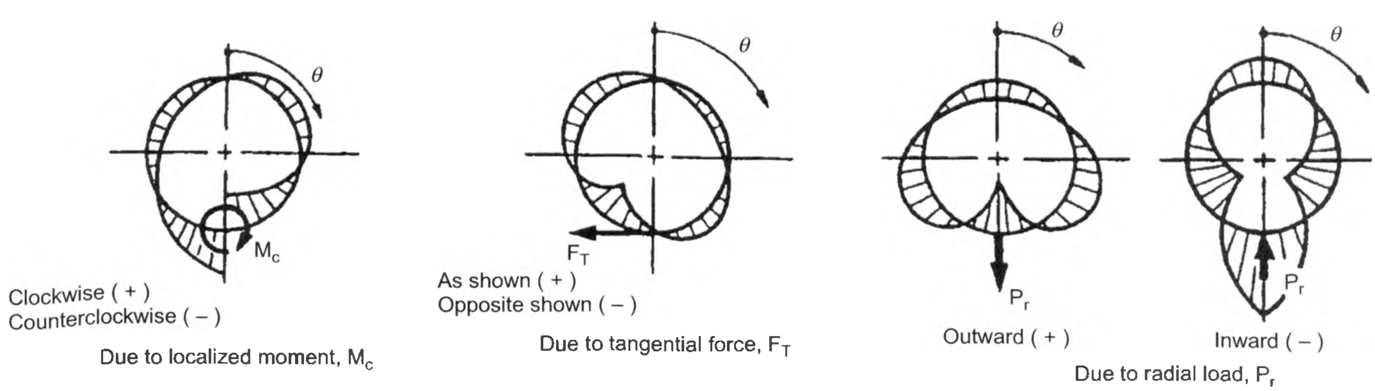
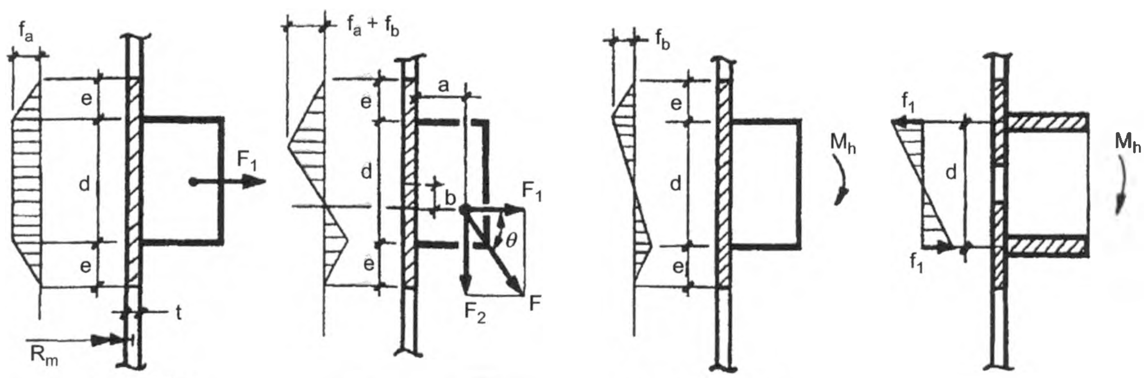


Figure 7-1. Moment diagrams for various ring loadings.

Table 7-1
Moments and forces in shell, M or T

Due to	Internal Moment, M	Tension/Compression Force, T
Circumferential moment, M_c	$M = \sum(K_m M_c)$	$T = \frac{\sum(C_m M_c)}{R_m}$
Tangential force, F_T	$M = \sum(K_T F_T)R_m$	$T = \sum(C_T F_T)$
Radial load, P_r	$M = \sum(K_r F_r)R_m$	$T = \sum(C_r F_r)$

Substitute R_1 for R_m if a ring is used. Values of K_m, K_T, K_r, C_m, C_T, and C_r are from Tables 7-4, 7-5, and 7-6.



Case 1

$e = 0.78\sqrt{R_m t}$

$f_a = \frac{F_1}{d + e}$

$f_1 = f_a$

Case 2

$e = 0.78\sqrt{R_m t}$

$F_1 = F\cos\theta$

$F_2 = F\sin\theta$

$M_h = aF_2 + bF_1$

$f_a = \frac{F_1}{d + e}$

$f_b = \frac{6M_h}{(d + e)(d + 2e)}$

$f_1 = f_a + f_b$

Case 3

$e = 0.78\sqrt{R_m t}$

$f_b = \frac{6M_h}{(d + e)(d + 2e)}$

$f_1 = f_b$

Case 4

Continuous rings

$f_1 = \frac{M_h}{d}$

Figure 7-2. Determination of radial load, f_1, for various shell loadings.

Allowable Stresses

Longitudinal tension: $<1.5SE =$
Longitudinal compression: Factor "B" $=$
Circumferential compression: $<0.5F_y =$
Circumferential buckling: p – lb/in.

$$p = \frac{3E_1 I}{4R^3}$$

(Assumes 4:1 safety factor)
Circumferential tension: $<1.5SE =$
Factor "B"

$$\frac{D_o}{t} = \quad = 0.05 \text{ min}$$

$$\frac{L}{D_o} = \quad = 50 \text{ max}$$

Enter Section II, Part D, Subpart 3, Fig. G, ASME Code

$A = \quad = 0.1$ max

Enter applicable material chart in ASME Code, Section II:

$B = \quad$ psi

For values of A falling to left of material line:

$$B = \frac{AE_1}{2}$$

Table 7-2
Shell stresses due to various loadings

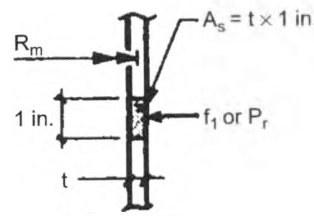
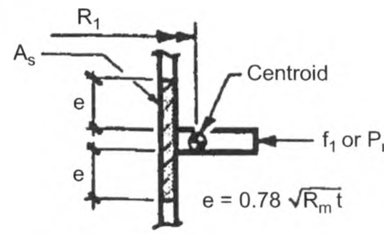
Stress Due To	Stress Direction	Without Stiffener	With Stiffener
			
Internal pressure, P	σ_x	$S_1 = \frac{PR_m}{2t}$	$S_1 = \frac{PR_m}{2t}$
	σ_ϕ	$S_2 = \frac{PR_m}{t}$	$S_2 = \frac{PR_m}{t}\left(\frac{A_s}{A_s + A_r}\right)$
Tension/compression force, T	σ_ϕ	$S_3 = \frac{T}{A_s}$	$S_3 = \frac{T}{A_s + A_r}$
		(+)tension (−)compression	(+)tension (−)compression
Local bending moment, M	σ_ϕ	$S_4 = \frac{6M}{t^2}$	$S_4 = \frac{M}{Z}$
		M can be (+) or (−)	M can be (+) or (−)
External pressure, P_e	σ_x	$S_5 = (-)\frac{P_e R_m}{2t}$	$S_5 = (-)\frac{P_e R_m}{2t}$
	σ_ϕ	$S_6 = (-)\frac{P_e R_m}{t}$	$S_6 = (-)\frac{2P_e R_m e}{A_s + A_r}$
Longitudinal moment, M_L	σ_x	$S_7 = \pm\frac{M_L}{\pi R_m^2 t}$	$S_7 = \pm\frac{M_L}{\pi R_m^2 t}$
Dead load, W	σ_x	$S_8 = (-)\frac{W}{2\pi R_m t}$	$S_8 = (-)\frac{W}{2\pi R_m t}$

Table 7-3
Combined stresses

Type	Tension	Compression
Longitudinal, σ_x	$\sigma_x = S_1 + S_7 - S_8$	$\sigma_x = (-)S_5 - S_7 - S_8$
Circumferential, σ_ϕ	$\sigma_\phi = S_2 + S_3 + S_4$	$\sigma_\phi = (-)S_3 - S_6 - S_4$

Procedure

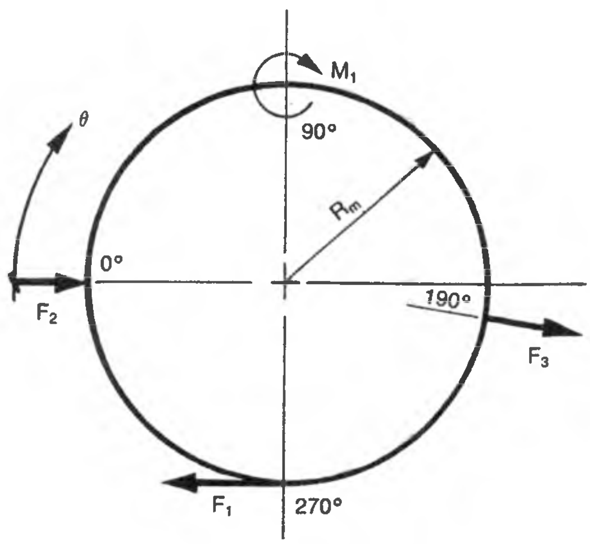
External localized loads (radial, moment, or tangential) produce internal bending moments, tension, and compression in ring sections. The magnitude of these moments and forces can be determined by this procedure, which consists essentially of the following steps:

1. Find moment or tension coefficients based on angular distances between applied loads, at each load from Tables 7–4, 7-5, and 7-6.
2. Superimpose the effects of various loadings by adding the product of coefficients times loads about any given point.

Notes

1. *Sign convention*: It is mandatory that sign convention be strictly followed to determine both the magnitude of the internal forces and tension or compression at any point.
   a. Coefficients in Tables 7-4, 7-5, and 7-6 are for angular distance θ measured between the point

EXAMPLE



GIVEN
$F_1 = (+)$ 1000 lb (would produce clockwise moment)
$F_2 = (-)$ 1500 lb (inward radial load)
$F_3 = (+)$ 500 lb (outward radial load)
$M_1 = (+)$ 800 in.-lb (would produce clockwise moment)
$R_m = 30$ in.
$t = 1$ in.
$A_s = 1$ in. $\times t = 1 \times 1 = 1$ in.^{2}
Since F_2 is the largest load assume the maximum moment will occur there.

Load	θ	Coefficient × Load (÷ Radius)	T
F_2	0°	$C_rF_2 = +0.2387\ (-1{,}500) =$	− 358.1
M_1	90°	$C_mM_1/R_m = -0.3183\ (+800)/30 =$	− 8.5
F_3	190°	$C_rF_3 = -0.2303\ (+500) =$	− 115.2
F_1	270°	$C_TF_1 = 0.0796\ (-1{,}000) =$	− 79.6
		T =	− 561.4 lb

$$\sigma_\phi = \frac{T}{A_s} = \frac{-561.4}{1} = -561 \text{ psi (compression)}$$

Load	θ	Coefficient × Load (× Radius)	M
F_2	0°	$K_rF_2R_m = -0.2387\ (-1500)\ 30 =$	+ 10,742
M_1	90°	$K_mM_1 = -0.0683\ (+800) =$	− 55
F_3	190°	$K_rF_3R_m = -0.0760\ (+500)\ 30 =$	− 1140
F_1	270°	$K_TF_1R_m = -0.0113\ (+1000)\ 30 =$	− 339
		M =	+ 9208 in.-lb

$$\sigma_\phi = \frac{6M}{t^2} = \frac{6(+9208)}{1^2} = +55{,}248 \text{ psi}$$

No good!

$$Z = \frac{M}{S} = \frac{9208}{15000} = 0.614 \text{ in.}^3$$

Use this ring.

Figure 7-3. Sample ring section with various loadings.

Notes *(Cont)*

on the ring under consideration and loads. Signs shown are for θ measured in the clockwise direction only.

b. Signs of coefficients in Tables 7-4, 7-5, and 7-6 are for outward radial loads and clockwise tangential forces and moments. For loads and moments in the opposite direction either the sign of the load or the sign of the coefficient must be reversed.

2. In Figure 7–4 the coefficients have already been combined for the loadings shown. The loads must be of equal magnitude and equally spaced. Signs of coefficients K_r and C_r are given for loads in the direction shown. Either the sign of the load or the sign of the coefficient may be reversed for loads in the opposite direction.
3. The maximum moment normally occurs at the point of the largest load; however, for unevenly spaced or mixed loadings, moments or tension should be investigated at each load, i.e., five loads require five analyses.

(Continued)

Table 7-4
Values of coefficients

	Localized Moment, M_c		Tangential Force, F_T			Localized Moment, M_c		Tangential Force, F_T	
θ	K_m	C_m	K_T	C_T	θ	K_m	C_m	K_T	C_T
0	+0.5	0	0	−0.5	180°	0	0	0	0
5°	+0.4584	−0.0277	−0.0190	−0.4773	185°	+0.0139	+0.0277	−0.0069	−0.0208
10°	+0.4169	−0.0533	−0.0343	−0.4512	190°	+0.0275	+0.0553	−0.0137	−0.0442
15°	+0.3759	−0.0829	−0.0462	−0.4221	195°	+0.0407	+0.0824	−0.0201	−0.0608
20°	+0.3356	−0.1089	−0.0549	−0.3904	200°	+0.0533	+0.1089	−0.0261	−0.0794
25°	+0.2960	−0.1345	−0.0606	−0.3566	205°	+0.0651	+0.1345	−0.0345	−0.0966
30°	+0.2575	−0.1592	−0.0636	−0.3210	210°	+0.0758	+0.1592	−0.0361	−0.1120
35°	+0.2202	−0.1826	−0.0641	−0.2843	215°	+0.0854	+0.1826	−0.0399	−0.1253
40°	+0.1843	−0.2046	−0.0625	−0.2468	220°	+0.0935	+0.2046	−0.0428	−0.1363
45°	+0.1499	−0.2251	−0.0590	−0.2089	225°	+0.1001	+0.2251	−0.0446	−0.1447
50°	+0.1173	−0.2438	−0.0539	−0.1712	230°	+0.1050	+0.2438	−0.0453	−0.1502
55°	+0.0865	−0.2607	−0.0475	−0.1340	235°	+0.1080	+0.2607	−0.0449	−0.1528
60°	+0.0577	−0.2757	−0.0401	−0.0978	240°	+0.1090	+0.2757	−0.0433	−0.1522
65°	+0.0310	−0.2885	−0.0319	−0.0629	245°	+0.1080	+0.2885	−0.0405	−0.1484
70°	+0.0064	−0.2991	−0.0233	−0.0297	250°	+0.1047	+0.2991	−0.0366	−0.1413
75°	−0.0158	−0.3075	−0.0144	+0.0014	255°	+0.0991	+0.3075	−0.0347	−0.1308
80°	−0.0357	−0.3135	−0.0056	+0.0301	260°	+0.0913	+0.3135	−0.0257	−0.1170
85°	−0.0532	−0.3171	+0.0031	+0.0563	265°	+0.0810	+0.3171	−0.0189	−0.0999
90°	−0.0683	−0.3183	+0.0113	+0.0796	270°	+0.0683	+0.3183	−0.0113	−0.0796
95°	−0.0810	−0.3171	+0.0189	+0.0999	275°	+0.0532	+0.3171	−0.0031	−0.0563
100°	−0.0913	−0.3135	+0.0257	+0.1170	280°	+0.0357	+0.3135	+0.0056	−0.0301
105°	−0.0991	−0.3075	+0.0347	+0.1308	285°	+0.0158	+0.3075	+0.0144	−0.0014
110°	−0.1047	−0.2991	+0.0366	+0.1413	290°	−0.0064	+0.2991	+0.0233	+0.0297
115°	−0.1079	−0.2885	+0.0405	+0.1484	295°	−0.0310	+0.2885	+0.0319	+0.0629
120°	−0.1090	−0.2757	+0.0433	+0.1522	300°	−0.0577	+0.2757	+0.0401	+0.0978
125°	−0.1080	−0.2607	+0.0449	+0.1528	305°	−0.0865	+0.2607	+0.0475	+0.1340
130°	−0.1050	−0.2438	+0.0453	+0.1502	310°	−0.1173	+0.2438	+0.0539	+0.1712
135°	−0.1001	−0.2251	+0.0446	+0.1447	315°	−0.1499	+0.2251	+0.0590	+0.2089
140°	−0.0935	−0.2046	+0.0428	+0.1363	320°	−0.1843	+0.2046	+0.0625	+0.2468
145°	−0.0854	−0.1826	+0.0399	+0.1253	325°	−0.2202	+0.1826	+0.0641	+0.2843
150°	−0.0758	−0.1592	+0.0361	+0.1120	330°	−0.2575	+0.1592	+0.0636	+0.3210
155°	−0.0651	−0.1345	+0.0345	+0.0966	335°	−0.2960	+0.1345	+0.0606	+0.3566
160°	−0.0533	−0.1089	+0.0261	+0.0794	340°	−0.3356	+0.1089	+0.0549	+0.3904
165°	−0.0407	−0.0824	+0.0201	+0.0608	345°	−0.3759	+0.0829	+0.0462	+0.4221
170°	−0.0275	−0.0553	+0.0137	+0.0442	350°	−0.4169	+0.0533	+0.0343	+0.4512
175°	−0.0139	−0.0277	+0.0069	+0.0208	355°	−0.4584	+0.0277	+0.0190	+0.4773

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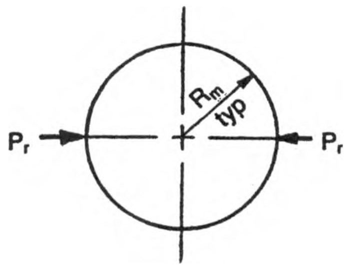
Table 7-5
Values of coefficient K_r due to outward radial load, P_r

θ	K_r	θ	K_r	θ	K_r	θ	K_r
0–360°	−0.2387	46–314	+0.0533	92–268	+0.0883	138–222	−0.0212
1–359	−0.2340	47–313	+0.0567	93–267	+0.0868	139–221	−0.0237
2–358	−0.2217	48–312	+0.0601	94–266	+0.0851	140–220	−0.0268
3–357	−0.2132	49–311	+0.0632	95–265	+0.0830	141–219	−0.0284
4–356	−0.2047	50–310	+0.0663	96–264	+0.0817	142–218	−0.0307
5–355	−0.1961	51–309	+0.0692	97–263	+0.0798	143–217	−0.0330
6–354	−0.1880	52–308	+0.0720	98–262	+0.0780	144–216	−0.0353
7–353	−0.1798	53–307	+0.0747	99–261	+0.0760	145–215	−0.0382
8–352	−0.1717	54–306	+0.0773	100–260	+0.0736	146–214	−0.0396
9–351	−0.1637	55–305	+0.0796	101–259	+0.0719	147–213	−0.0418
10–350	−0.1555	56–304	+0.0819	102–258	+0.0698	148–212	−0.0438
11–349	−0.1480	57–303	+0.0841	103–257	+0.0677	149–211	−0.0459
12–348	−0.1402	58–302	+0.0861	104–256	+0.0655	150–210	−0.0486
13–347	−0.1326	59–301	+0.0880	105–255	+0.0627	151–209	−0.0498
14–346	−0.1251	60–300	+0.0897	106–254	+0.0609	152–208	−0.0517
15–345	−0.1174	61–299	+0.0914	107–253	+0.0586	153–207	−0.0535
16–344	−0.1103	62–298	+0.0940	108–252	+0.0562	154–206	−0.0553
17–343	−0.1031	63–297	+0.0944	109–251	+0.0538	155–205	−0.0577
18–342	−0.0960	64–296	+0.0957	110–250	+0.0508	156–204	−0.0586
19–341	−0.0890	65–295	+0.0967	111–249	+0.0489	157–203	−0.0602
20–340	−0.0819	66–294	+0.0979	112–248	+0.0464	158–202	−0.0617
21–339	−0.0754	67–293	+0.0988	113–247	+0.0439	159–201	−0.0633
22–338	−0.0687	68–292	+0.0997	114–246	+0.0431	160–200	−0.0654
23–337	−0.0622	69–291	+0.1004	115–245	+0.0381	161–199	−0.0660
24–336	−0.0558	70–290	+0.1008	116–244	+0.0361	162–198	−0.0673
25–335	−0.0493	71–289	+0.1014	117–243	+0.0335	163–197	−0.0686
26–334	−0.0433	72–288	+0.1018	118–242	+0.0309	164–196	−0.0697
27–333	−0.0373	73–287	+0.1019	119–241	+0.0283	165–195	−0.0715
28–332	−0.0314	74–286	+0.1020	120–240	+0.0250	166–194	−0.0719
29–331	−0.0256	75–285	+0.1020	121–239	+0.0230	167–193	−0.0728
30–330	−0.0197	76–284	+0.1020	122–238	+0.0203	168–192	−0.0737
31–329	−0.0144	77–283	+0.1019	123–237	+0.0176	169–191	−0.0746
32–328	−0.0089	78–282	+0.1017	124–236	+0.0145	170–190	−0.0760
33–327	−0.0037	79–281	+0.1013	125–235	+0.0116	171–189	−0.0764
34–326	+0.0015	80–280	+0.1006	126–234	+0.0090	172–188	−0.0768
35–325	+0.0067	81–279	+0.1003	127–233	+0.0070	173–187	−0.0772
36–324	+0.0115	82–278	+0.0997	128–232	+0.0044	174–186	−0.0776
37–323	+0.0162	83–277	+0.0989	129–231	+0.0017	175–185	−0.0787
38–322	+0.0209	84–276	+0.0981	130–230	−0.0016	176–184	−0.0789
39–321	+0.0254	85–275	+0.0968	131–229	−0.0035	177–183	−0.0791
40–320	+0.0299	86–274	+0.0961	132–228	−0.0061	178–182	−0.0793
41–319	+0.0340	87–273	+0.0950	133–227	−0.0087	179–181	−0.0795
42–318	+0.0381	88–272	+0.0938	134–226	−0.0113	180	−0.0796
43–317	+0.0421	89–271	+0.0926	135–225	−0.0145		
44–316	+0.0460	90–270	+0.0909	136–224	−0.0163		
45–315	+0.0497	91–269	+0.0898	137–223	−0.0188		

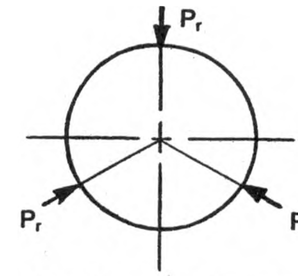
Table 7-6
Values of coefficient C_r due to radial load, P_r

θ	C_r	θ	C_r	θ	C_r	θ	C_r	θ	C_r	θ	C_r
0–360°	+0.2387	31–329	+0.4175	62–298	+0.4010	93–267	+0.2280	124–236	−0.0040	155–205	−0.1870
1–359	+0.2460	32–328	+0.4200	63–297	+0.3975	94–266	+0.2225	125–235	−0.0018	156–204	−0.1915
2–358	+0.2555	33–327	+0.4225	64–296	+0.3945	95–265	+0.2144	126–234	−0.0175	157–203	−0.1945
3–357	+0.2650	34–326	+0.4250	65–295	+0.3904	96–264	+0.2075	127–233	−0.0250	158–202	−0.1985
4–356	+0.2775	35–325	+0.4266	66–294	+0.3875	97–263	+0.2000	128–232	−0.0325	159–201	−0.2025
5–355	+0.2802	36–324	+0.4280	67–293	+0.3830	98–262	+0.1925	129–231	−0.0400	160–200	−0.2053
6–354	+0.2870	37–323	+0.4300	68–292	+0.3790	99–261	+0.1850	130–230	−0.0471	161–199	−0.2075
7–353	+0.2960	38–322	+0.4315	69–291	+0.3740	100–260	+0.1774	131–229	−0.0550	162–198	−0.2110
8–352	+0.3040	39–321	+0.4325	70–290	+0.3688	101–259	+0.1700	132–228	−0.0620	163–197	−0.2140
9–351	+0.3100	40–320	+0.4328	71–289	+0.3625	102–258	+0.1625	133–227	−0.0675	164–196	−0.2170
10–350	+0.3171	41–319	+0.4330	72–288	+0.3600	103–257	+0.1550	134–226	−0.0750	165–195	−0.2198
11–349	+0.3240	42–318	+0.4332	73–287	+0.3540	104–256	+0.1480	135–225	−0.0804	166–194	−0.2220
12–348	+0.3310	43–317	+0.4335	74–286	+0.3490	105–255	+0.1394	136–224	−0.0870	167–193	−0.2240
13–347	+0.3375	44–316	+0.4337	75–285	+0.3435	106–254	+0.1400	137–223	−0.0940	168–192	−0.2260
14–346	+0.3435	45–315	+0.4340	76–284	+0.3381	107–253	+0.1300	138–222	−0.1000	169–191	−0.2280
15–345	+0.3492	46–314	+0.4332	77–283	+0.3325	108–252	+0.1150	139–221	−0.1050	170–190	−0.2303
16–344	+0.3550	47–313	+0.4324	78–282	+0.3270	109–251	+0.1075	140–220	−0.1115	171–189	−0.2315
17–343	+0.3600	48–312	+0.4316	79–281	+0.3200	110–250	+0.1011	141–219	−0.1170	172–188	−0.2325
18–342	+0.3655	49–311	+0.4308	80–280	+0.3150	111–249	+0.0925	142–218	−0.1230	173–187	−0.2345
19–341	+0.3720	50–310	+0.4301	81–279	+0.3090	112–248	+0.0840	143–217	−0.1280	174–186	−0.2351
20–340	+0.3763	51–309	+0.4283	82–278	+0.3025	113–247	+0.0760	144–216	−0.1350	175–185	−0.2366
21–339	+0.3810	52–308	+0.4266	83–277	+0.2960	114–246	+0.0700	145–215	−0.1398	176–184	−0.2370
22–338	+0.3855	53–307	+0.4248	84–276	+0.2900	115–245	+0.0627	146–214	−0.1450	177–183	−0.2375
23–337	+0.3900	54–306	+0.4231	85–275	+0.2837	116–244	+0.0550	147–213	−0.1500	178–182	−0.2380
24–336	+0.3940	55–305	+0.4214	86–274	+0.2775	117–243	+0.0490	148–212	−0.1550	179–181	−0.2384
25–335	+0.3983	56–304	+0.4180	87–273	+0.2710	118–242	+0.0400	149–211	−0.1605	180	−0.2387
26–334	+0.4025	57–303	+0.4160	88–272	+0.2650	119–241	+0.0335	150–210	−0.1651		
27–333	+0.4060	58–302	+0.4130	89–271	+0.2560	120–240	+0.0250	151–209	−0.1690		
28–332	+0.4100	59–301	+0.4100	90–270	+0.2500	121–239	+0.0175	152–208	−0.1745		
29–331	+0.4125	60–300	+0.4080	91–269	+0.2430	122–238	+0.0105	153–207	−0.1780		
30–330	+0.4151	61–299	+0.4040	92–268	+0.2360	123–237	+0.0025	154–206	−0.1825		



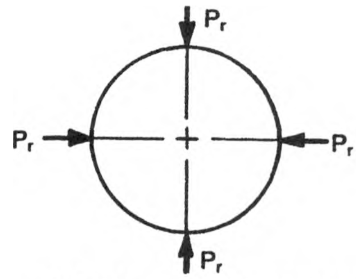
At loads		Between loads	
K_r	+ 0.3183	K_r	− 0.1817
C_r	+ 0	C_r	− 0.5

Case 1



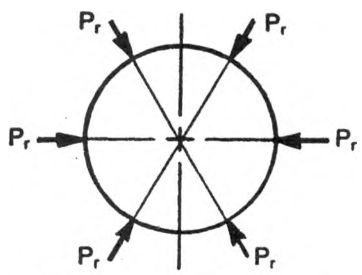
At loads		Between loads	
K_r	+ 0.1888	K_r	− 0.1
C_r	− 0.2887	C_r	− 0.5773

Case 2



At loads		Between loads	
K_r	+ 0.1366	K_r	− 0.0705
C_r	− 0.5	C_r	− 0.707

Case 3



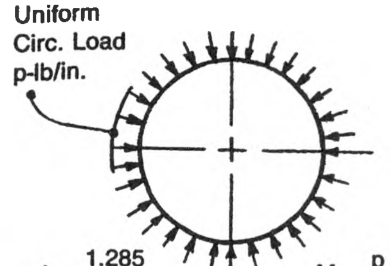
At loads		Between loads	
K_r	+ 0.0889	K_r	− 0.045
C_r	− 0.866	C_r	− 1.0

Case 4

For any number of equally spaced loads
ϕ = ½ angle between loads, radians

- *At loads:* $K_r = 0.5\left[\frac{1}{\phi} - \frac{\cos\phi}{\sin\phi}\right]$
- *Between loads:* $K_r = -0.5\left[\frac{1}{\sin\phi} - \frac{1}{\phi}\right]$
- *Tension force, T:* $T = \frac{P_r}{2}\left[\frac{1}{\sin\phi}\right]$

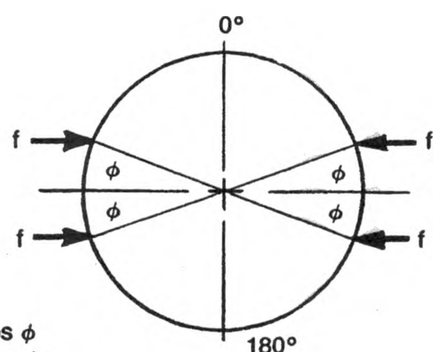
Case 5



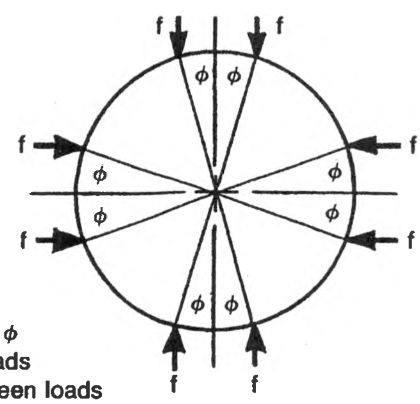
$$\lambda = \frac{1.285}{\sqrt{R_m t}} \qquad M = \frac{p}{4\lambda}$$

$$\sigma_x = \frac{-3p}{2\lambda t^2} \qquad \sigma_\phi = \frac{-pR_m\lambda}{2t}$$

Case 6



$P_r = f\cos\phi$
K_r is at loads
C_r is between loads



$P_r = f\cos\phi$
K_r is at loads
C_r is between loads

ϕ	K_r	C_r	ϕ	K_r	C_r
1°	0.6185	− 1.0	10°	0.4656	− 0.985
2°	0.6011	− 0.999	15°	0.3866	− 0.966
3°	0.5836	− 0.998	20°	0.3152	− 0.940
4°	0.5663	− 0.997	25°	0.2536	− 0.906
5°	0.5498	− 0.996	30°	0.2036	− 0.866
6°	0.5319	− 0.995	35°	0.1668	− 0.819
7°	0.5150	− 0.992	40°	0.1441	− 0.766
8°	0.4980	− 0.990	45°	0.1366	− 0.707
9°	0.4813	− 0.986			

Case 7

ϕ	K_r	C_r	ϕ	K_r	C_r
1°	0.2540	− 1.411	10°	0.1302	− 1.393
2°	0.2375	− 1.410	15°	0.0902	− 1.366
3°	0.2214	− 1.409	20°	0.0688	− 1.329
4°	0.2062	− 1.408	25°	0.0688	− 1.282
5°	0.1918	− 1.407	30°	0.0902	− 1.225
6°	0.1780	− 1.406	35°	0.1324	− 1.158
7°	0.1649	− 1.405	40°	0.1939	− 1.083
8°	0.1525	− 1.404	45°	0.2732	− 1.00
9°	0.1409	− 1.397			

Case 8

Figure 7-4. Values of coefficients K_r and C_r for various loadings.

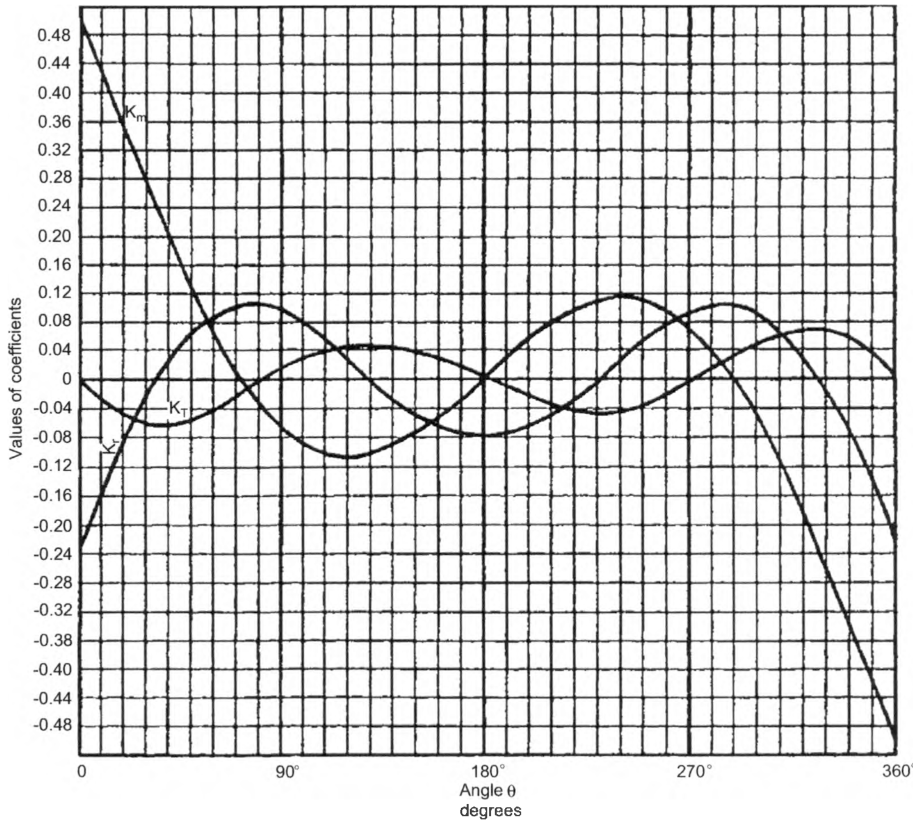


Figure 7-5. Graph of internal moment coefficients K_m, K_r, and K_T.

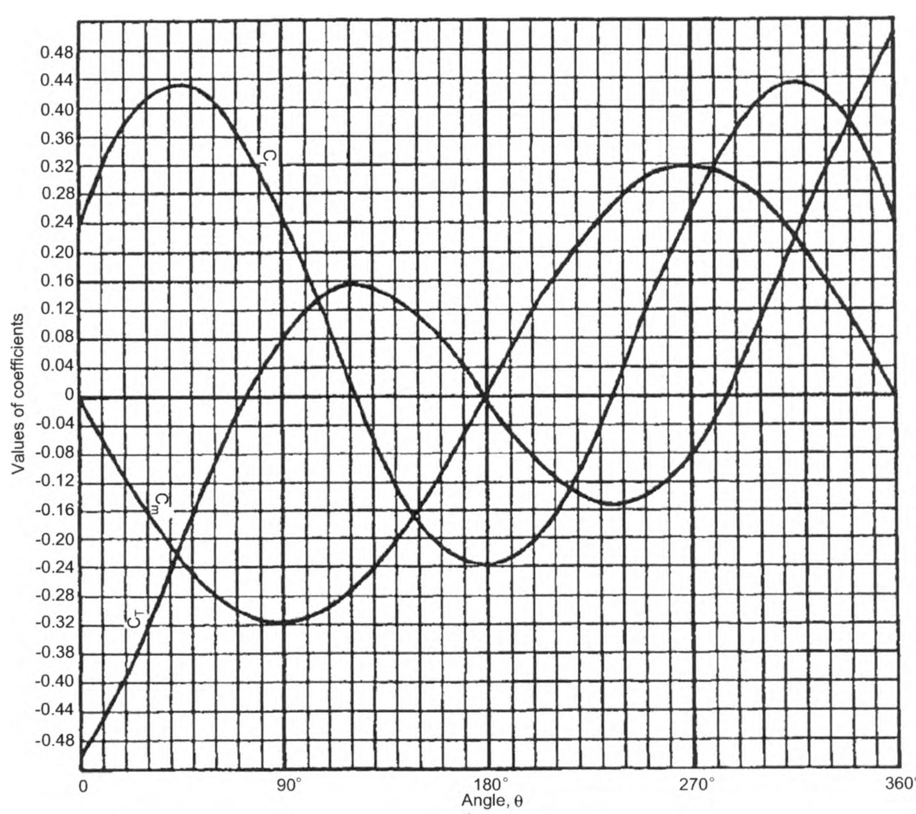


Figure 7-6. Graph of circumferential tension/compression coefficients C_m, C_r, and C_T.

Notes *(Cont)*

4. This procedure uses strain-energy concepts.
5. The following is assumed.
   a. Rings are of uniform cross section.
   b. Material is elastic, but is not stressed beyond elastic limit.
   c. Deformation is caused mainly by bending.
   d. All loads are in the same plane.
   e. The ring is not restrained and is supported along its circumference by a number of equidistant simple supports (therefore conservative for use on cylinders).
   f. The ring is of such large radius in comparison with its radial thickness that the deflection theory for straight beams is applicable.

Procedure 7-2: Design of Partial Ring Stiffeners [7]

Notation

M_L = longitudinal moment, in.-lb
M = internal bending moment, shell, in.-lb
F_b = allowable bending stress, psi
f_b = bending stress, psi
f or f_n = concentrated loads on stiffener due to radial or moment load on clip, lb
F_x = function or moment coefficient (see Table 7-7) = $e^{-\beta x}(\cos \beta x - \sin \beta x)$
E_v = modulus of elasticity of vessel shell at design temperature, psi
E_s = modulus of elasticity of stiffener at design temperature, psi
e = log base 2.71
I = moment of inertia of stiffener, $in.^4$
Z = section modulus of stiffener, $in.^3$
K = "spring constant" or "foundation modulus", $lb/in.^3$
x = distance between loads, in.
β = damping factor, dimensionless
P_r = radial load, lb

Table 7-7
Values Of Function F_x

βx	F_x	βx	F_x
0	1.0	0.55	0.1903
0.05	0.9025	0.6	0.1431
0.1	0.8100	0.65	0.0997
0.15	0.7224	0.7	0.0599
0.2	0.6398	0.75	0.0237
0.25	0.5619	0.8	(−)0.0093
0.3	0.4888	0.85	(−)0.0390
0.35	0.4203	0.9	(−)0.0657
0.4	0.3564	0.95	(−)0.0896
0.45	0.2968	1.0	(−)0.1108
0.5	0.2415		

Formulas

1. *Single load.* Determine concentrated load on each stiffener depending on whether there is a radial load or moment loading, single or double stiffener.

$$f =$$

- *Calculate foundation modulus, K.*

$$K = \frac{E_v t}{R^2}$$

- *Assume stiffener size and calculate Z and I.*
  Proposed size: _____

$$I = \frac{bh^3}{12}$$

$$Z = \frac{bh^2}{6}$$

- *Calculate damping factor β based on proposed stiffener size.*

$$\beta = \sqrt[4]{\frac{K}{4E_s I}}$$

- *Calculate internal bending moment in stiffener, M.*

$$M = \frac{f}{4\beta}$$

- *Calculate bending stress, f_b.*

$$f_b = \frac{M}{Z}$$

If bending stress exceeds allowable ($F_b = 0.6F_y$), increase size of stiffener and recalculate I, Z, β, M, and f_b.

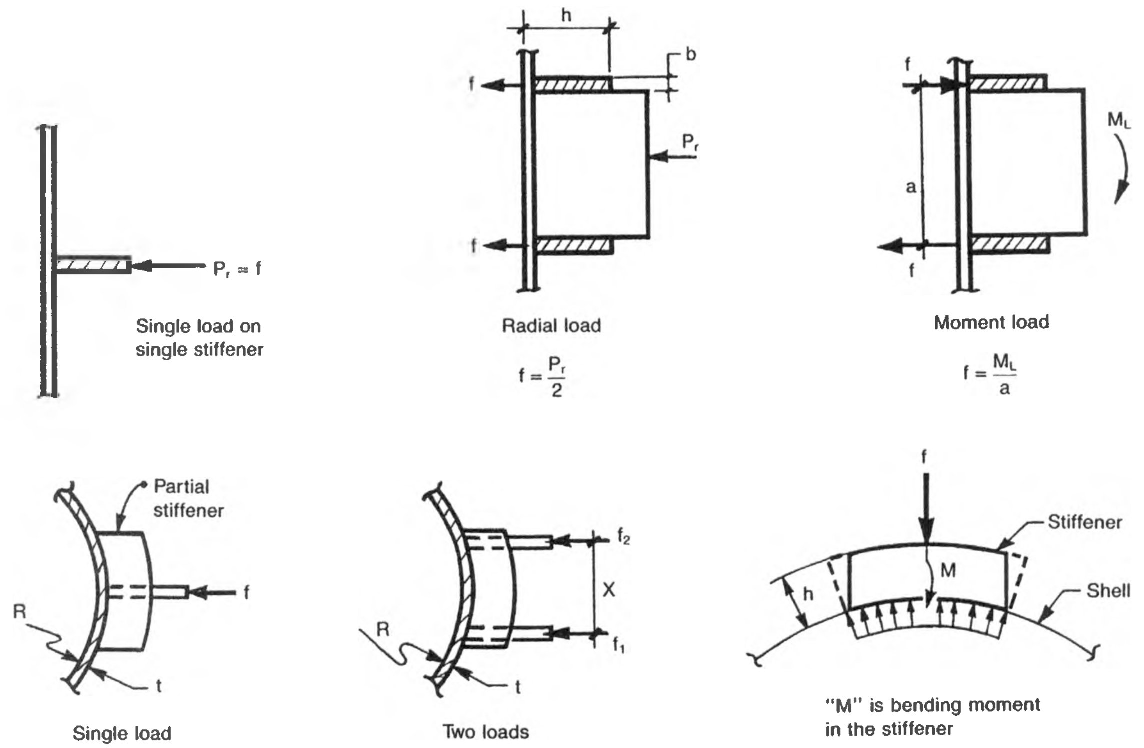


Figure 7-7. Dimensions, forces, and loadings for partial ring stiffeners.

2. *Multiple loads (see Figure 7-8).* Determine concentrated loads on stiffener(s). Loads must be of equal magnitude.

$$f = f_1 = f_2 = \ldots = f_n$$

- *Calculate foundation modulus, K.*

$$K = \frac{E_y t}{R^2}$$

- *Assume a stiffener size and calculate I and Z.*

Proposed size: _____

$$I = \frac{bh^3}{12}$$

$$Z = \frac{bh^2}{6}$$

- *Calculate damping factor β based on proposed stiffener size.*

$$\beta = \sqrt[4]{\frac{K}{4E_s I}}$$

- *Calculate internal bending moment in stiffener.*

Step 1: Determine βx for each load (βx is in radians).
Step 2: Determine F_x for each load from Table 7-7 or calculate as follows:

$$F_x = e^{-\beta x}(\cos\beta x - \sin\beta x)$$

Step 3: Calculate bending moment, M

$\beta x_0 = 0$	$F_1 = 1$
$\beta x_1 =$ __	$F_2 =$ __
$\beta x_2 =$ __	$F_3 =$ __
$\beta x_n =$ __	$F_n =$ __
	$\sum F_x =$ __

$$M = \frac{f}{4\beta}\left(\sum F_x\right)$$

- *Calculate bending stress, f_b.*

$$f_b = \frac{M}{Z}$$

Notes

1. This procedure is based on the beam-on-elastic-foundation theory. The elastic foundation is the vessel shell and the beam is the partial ring stiffener. The stiffener must be designed to be stiff enough to transmit the load(s) uniformly over its full length.

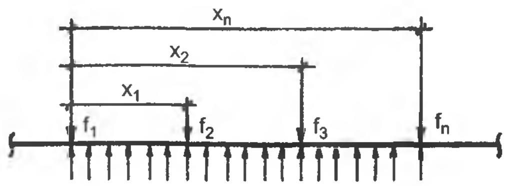


Figure 7-8. Dimensions and loading diagram for beam on elastic foundation analysis.

The flexibility of the vessel shell is taken into account. The length of the vessel must be at least 4.9 $\sqrt{Rt}$ to qualify for the infinitely long beam theory.

2. The case of multiple loads uses the principle of superposition. That is, the effect of each load may be determined independent of the other loads and the total effect may be determined by adding the individual effects.
3. This procedure determines the bending stress in the stiffener only. The stresses in the vessel shell should be checked by an appropriate local load procedure. These local stresses are secondary bending stresses and should be combined with *primary* membrane and bending stresses.

Procedure 7-3: Attachment Parameters

This procedure is for use in converting the area of attachments into shapes that can readily be applied in design procedures. Irregular attachments (not round, square, or rectangular) can be converted into a rectangle which has:

- The same moment of inertia
- The same ratio of length to width of the original attachment

In addition, a rectangular load area may be reduced to an "equivalent" square area.

Bijlaard recommends, for non-rectangular attachments, the loaded rectangle can be assumed to be that which has the same moment of inertia with respect to the moment axis as the plan of the actual attachment. Further, it should be assumed that the dimensions of the rectangle in the longitudinal and circumferential directions have the same ratio as the two dimensions of the attachment in these directions.

Dodge comments on this method in WRC Bulletin 198: "Although the 'equivalent moment of inertia procedure' is simple and direct, it was not derived by any mathematical or logical reasoning which would allow the designer to rationalize the accuracy of the results."

Dodge goes on to recommend an alternative procedure based on the principle of superposition. This method would divide irregular attachments into a composite of one or more rectangular sub-areas.

Neither method is entirely satisfactory and each ignores the effect of local stiffness provided by the attachment's shape. An empirical method should take into consideration the "area of influence" of the attachment which would account for the attenuation length or decay length of the stress in question.

Studies by Roark would indicate short zones of influence in the longitudinal direction (quick decay) and a much broader area of influence in the circumferential direction (slow decay, larger attenuation). This would also seem to account for the attachment and shell acting as a unit, which they of course do.

Since no hard and fast rules have yet been determined, it would seem reasonable to apply the factors as outlined in this procedure for general applications. Very large or critical loads should, however, be examined in depth.

Notes

1. $b = t_c + 2t_w + 2t_s$ where t_w = fillet weld size and t_s = thickness of shell.
2. Clips must be closer than $\sqrt{Rt}$ if running circumferentially or closer than 6 in. if running longitudinally to be considered as a single attachment.

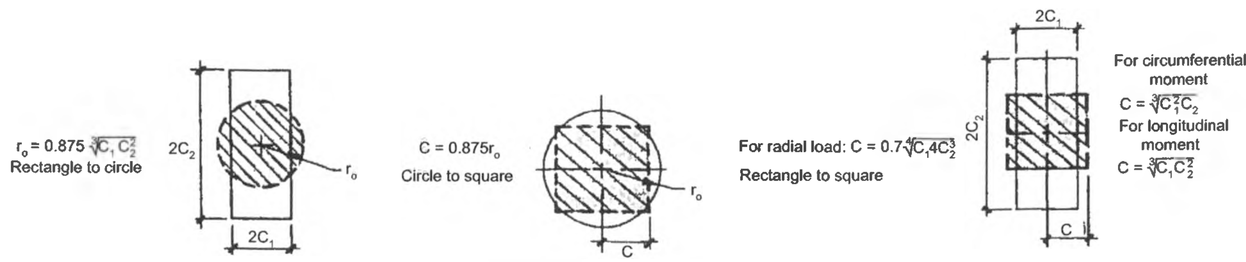
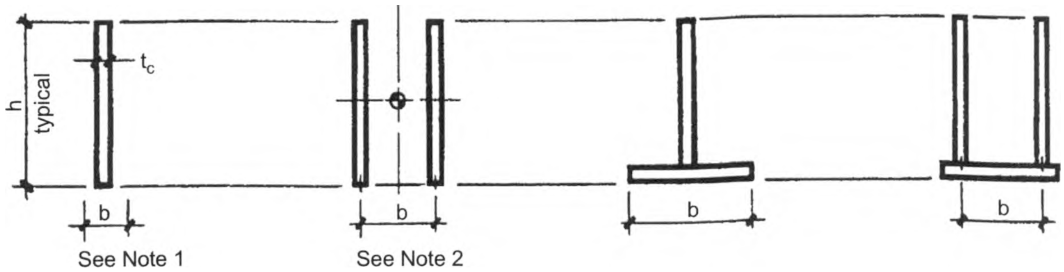
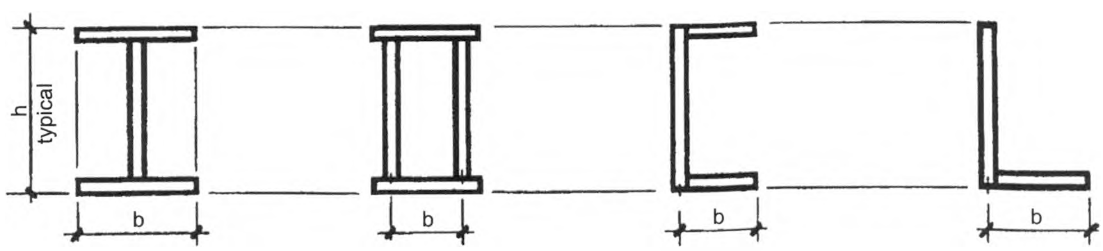


Figure 7-9. Attachment parameters for solid attachments.



C_1	0.5b	0.3b	0.25b	0.3b
C_2	0.4h	0.4h	0.4h	0.4h



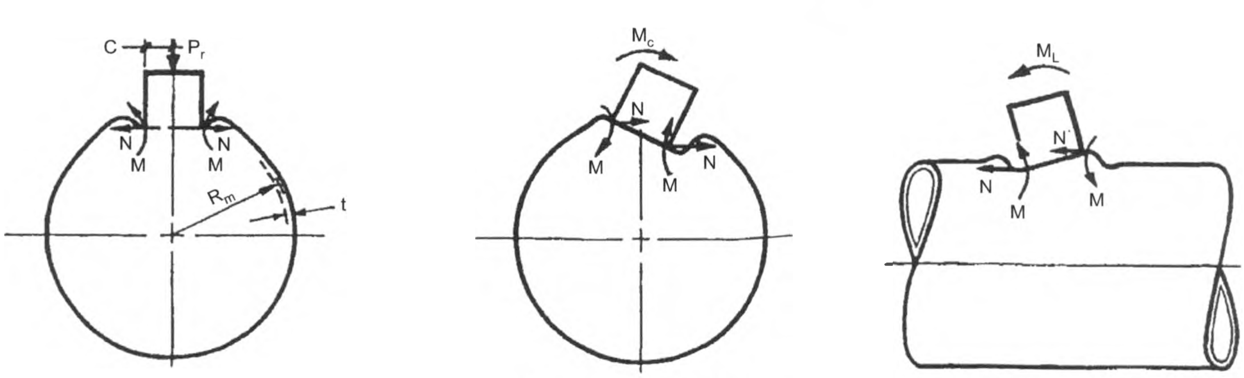
C_1	0.4b	0.5b	0.3b	0.2b
C_2	0.5h	0.5h	0.5h	0.4h

Figure 7-10. Attachment parameters for nonsolid attachments.

Procedure 7-4: Stresses in Cylindrical Shells from External Local Loads [7,9,10,11]

Notation

P_r = radial load, lb
P = internal design pressure, psi
M_L = external longitudinal moment, in.-lb
M_c = external circumferential moment, in.-lb
M_T = external torsional moment, in.-lb
M_x = internal circumferential moment, in.-lb/in.
M_ϕ = internal longitudinal moment, in.-lb/in.
V_L = longitudinal shear force, lb
V_c = circumferential shear force, lb
R_m = mean radius of shell, in.
r_o = outside radius of circular attachment, in.
r = corner radius of attachment, in.
K_n, K_b = stress concentration factors



Radial load–membrane stress is compressive for inward radial load and tensile for outward load

Circumferential moment

Longitudinal moment

Figure 7-11. Loadings and forces at local attachments in cylindrical shells.

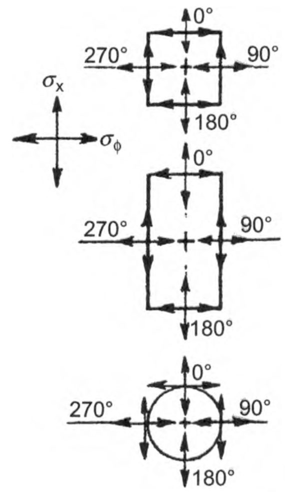


Figure 7-12. Stress indices of local attachments.

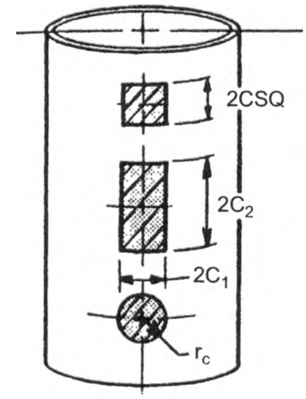
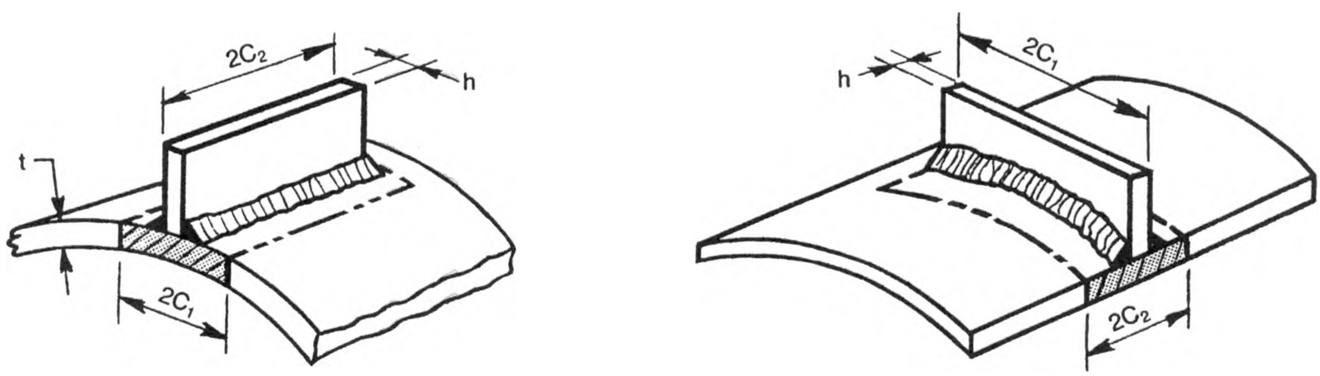


Figure 7-13. Load areas of local attachments. For circular attachments use $C = 0.875r_o$.



$2C_1 = h + 2w + 2t$
w = leg of fillet weld
h = thickness of attachment

$2C_2 = h + 2w + 2t$
Note: Only ratios of C_1/C_2 between 0.25 and 4 may be computed by this procedure.

Figure 7-14. Dimensions for clips and attachments.

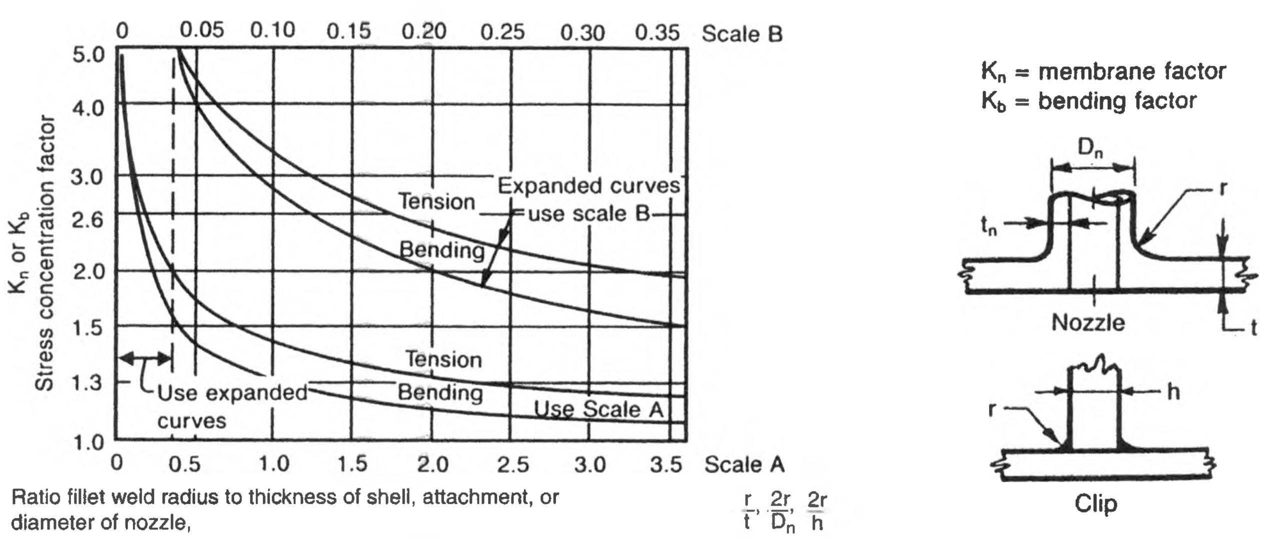
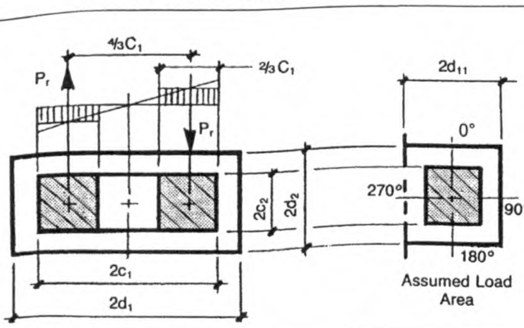


Figure 7-15. Stress concentration factors. (*Reprinted by permission of the Welding Research Council.*)

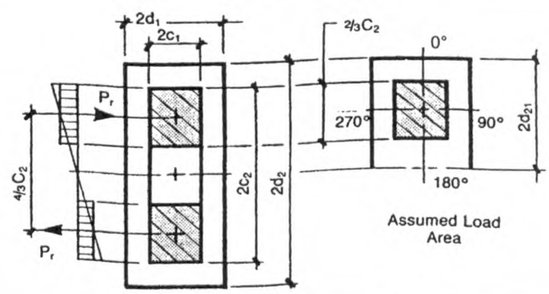
K_c, K_L, K_1, K_2 = coefficients to determine β for rectangular attachments
N_x = membrane force in shell, longitudinal, lb/in.
N_ϕ = membrane force in shell, circumferential, lb/in.
τ_T = torsional shear stress, psi
τ_s = direct shear stress, psi
σ_x = longitudinal normal stress, psi
σ_ϕ = circumferential normal stress, psi
C = one-half width of square attachment, in.
C_c, C_L = multiplication factors for rectangular attachments
C_1 = one-half circumferential width of a rectangular attachment, in.
C_2 = one-half longitudinal length of a rectangular attachment, in.
h = thickness of attachment, in.
d_n = outside diameter of circular attachment, in.
t_e = equivalent thickness of shell and reinforcing, in.
t_p = thickness of reinforcing pad, in.
t = shell thickness, in.
$\gamma, \beta, \beta_1, \beta_2$ = ratios based on vessel and attachment geometry

COMPUTING GEOMETRIC PARAMETERS FOR LOADS ON ATTACHMENTS WITH REINFORCING PADS

CIRCUMFERENTIAL MOMENT



LONGITUDINAL MOMENT



Circumferential Moment: At Edge of Attachment		Circumferential Moment: At Edge of Pad		Longitudinal Moment: At Edge of Attachment		Longitudinal Moment: At Edge of Pad	
$R_m = \frac{I.D. + t + t_p}{2}$		$R_m = \frac{I.D. + t}{2}$		$R_m = \frac{I.D. + t + t_p}{2}$		$R_m = \frac{I.D. + t}{2}$	
$t_e = \sqrt{t^2 + t_p^2}$		t		$t_e = \sqrt{t^2 + t_p^2}$		t	
$\gamma = \frac{R_m}{t_e}$		$\gamma = \frac{R_m}{t}$		$\gamma = \frac{R_m}{t_e}$		$\gamma = \frac{R_m}{t}$	
$C_1 = \frac{2C_1}{6}$		$C_1 = \frac{2d_{11}}{2}$		$C_1 = \frac{2C_1}{2}$		$C_1 = \frac{2d_1}{2}$	
$C_2 = \frac{2C_2}{2}$		$C_2 = \frac{2d_2}{2}$		$C_2 = \frac{2C_2}{6}$		$C_2 = \frac{2d_{21}}{2}$	
$\beta_1 = \frac{C_1}{R_m}$		$\beta_1 = \frac{C_1}{R_m}$		$\beta_1 = \frac{C_1}{R_m}$		$\beta_1 = \frac{C_1}{R_m}$	
$\beta_2 = \frac{C_2}{R_m}$		$\beta_2 = \frac{C_2}{R_m}$		$\beta_2 = \frac{C_2}{R_m}$		$\beta_2 = \frac{C_2}{R_m}$	
$\frac{\beta_1}{\beta_2}$		$\frac{\beta_1}{\beta_2}$		$\frac{\beta_1}{\beta_2}$		$\frac{\beta_1}{\beta_2}$	
β for N_ϕ		β for N_ϕ		β for N_ϕ		β for N_ϕ	
β for N_x		β for N_x		β for N_x		β for N_x	
β for M_ϕ		β for M_ϕ		β for M_ϕ		β for M_ϕ	
β for M_x		β for M_x		β for M_x		β for M_x	

Geometric Parameters

$$\gamma = \frac{R_m}{t}$$

$$\beta = \frac{C}{R_m}$$

or for circular attachments:

$$\frac{0.875r_o}{R_m}$$

For rectangular attachments:

$$\beta_1 = \frac{C_1}{R_m}$$

$$\beta_2 = \frac{C_2}{R_m}$$

Procedure

To calculate stresses due to radial load P_r, longitudinal moment M_L, and circumferential moment M_c, on a cylindrical vessel, follow the following steps:

Step 1: Calculate geometric parameters:

a. Round attachments:

$$\gamma = \frac{R_m}{t}$$

$$\beta = \frac{0.875r_o}{R_m}$$

b. Square attachments:

$$\gamma = \frac{R_m}{t}$$

$$\beta = \frac{C}{R_m}$$

c. Rectangular attachments:

$$\gamma = \frac{R_m}{t}$$

β values for radial load, longitudinal moment, and circumferential moment vary based on ratios of β_1/β_2. Follow procedures that follow these steps to find β values.

Step 2: Using γ and β values; from Step 1, enter applicable graphs, Figures 7-21 through 7-26 to dimensionless membrane forces and bending moments in shell.

Step 3: Enter values obtained from Figures 7-21 through 7-26 into Table 7-11 and compute stresses.

Step 4: Enter stresses computed in Table 7-11 for various load conditions in Table 7-12. Combine stresses in accordance with sign convention of Table 7-12.

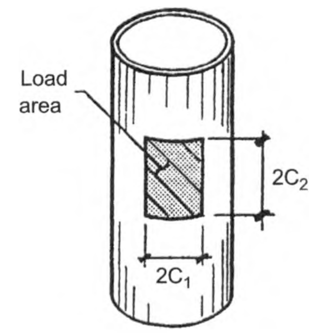


Figure 7-16. Dimensions of load areas.

Computing β Values for Rectangular Attachments

$$\beta_1 = \frac{C_1}{R_m}$$

$$\beta_2 = \frac{C_2}{R_m}$$

$$\frac{\beta_1}{\beta_2}$$

β Values for Radial Load

From Table 7-8 select values of K_1 and K_2 and compute four β values as follows:

If $\frac{\beta_1}{\beta_2} \geq 1$, then β

$$= \left[1 - \frac{1}{3}\left(\frac{\beta_1}{\beta_2} - 1\right)(1 - K_1)\right]\sqrt{\beta_1\beta_2}$$

Table 7-8
β Values of radial loads

	K_1	K_2	β
N_ϕ	0.91	1.48	
N_x	1.68	1.2	
M_ϕ	1.76	0.88	
M_x	1.2	1.25	

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If $\frac{\beta_1}{\beta_2} < 1$, then β

$$= \left[1 - \frac{4}{3}\left(1 - \frac{\beta_1}{\beta_2}\right)(1 - K_2)\right]\sqrt{\beta_1\beta_2}$$

β Values for Longitudinal Moment

From Table 7-9 select values of C_L and K_L and compute values of β as follows:

For N_x and N_ϕ, $\beta = \sqrt[3]{\beta_1\beta_2^2}$

For M_ϕ, $\beta = K_L\sqrt[3]{\beta_1\beta_2^2}$

For M_x, $\beta = K_L\sqrt[3]{\beta_1\beta_2^2}$

	C_L	K_L	β
N_ϕ			
N_x			
M_ϕ			
M_x			

Table 7-9
Coefficients for longitudinal moment, M_L

β_1/β_2	γ	C_L for N_ϕ	C_L for N_x	K_L for M_ϕ	K_L for M_x
	15	0.75	0.43	1.80	1.24
	50	0.77	0.33	1.65	1.16
0.25	100	0.80	0.24	1.59	1.11
	200	0.85	0.10	1.58	1.11
	300	0.90	0.07	1.56	1.11
	15	0.90	0.76	1.08	1.04
	50	0.93	0.73	1.07	1.03
0.5	100	0.97	0.68	1.06	1.02
	200	0.99	0.64	1.05	1.02
	300	1.10	0.60	1.05	1.02
	15	0.89	1.00	1.01	1.08
	50	0.89	0.96	1.00	1.07
1	100	0.89	0.92	0.98	1.05
	200	0.89	0.99	0.95	1.01
	300	0.95	1.05	0.92	0.96
	15	0.87	1.30	0.94	1.12
	50	0.84	1.23	0.92	1.10
2	100	0.81	1.15	0.89	1.07
	200	0.80	1.33	0.84	0.99
	300	0.80	1.50	0.79	0.91
	15	0.68	1.20	0.90	1.24
	50	0.61	1.13	0.86	1.19
4	100	0.51	1.03	0.81	1.12
	200	0.50	1.18	0.73	0.98
	300	0.50	1.33	0.64	0.83

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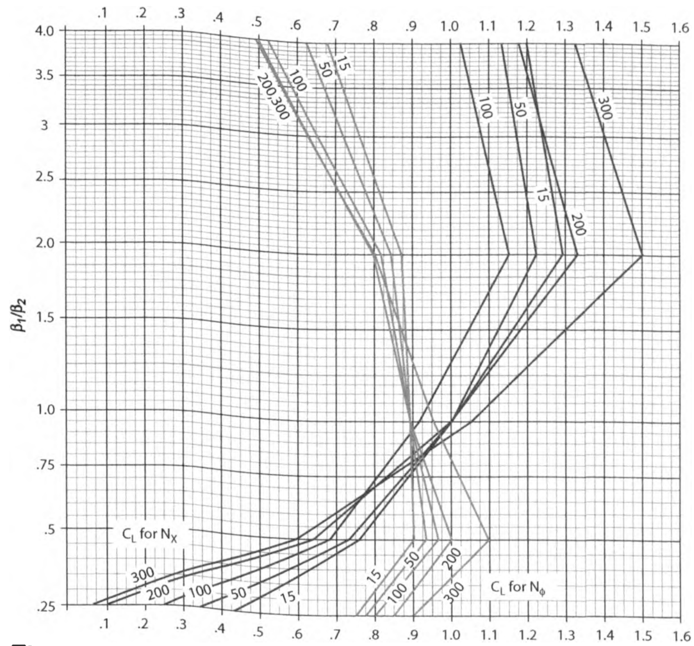

Figure 7-17. Graph of coefficients C_L for values N_ϕ & N_X from Table 7-9.

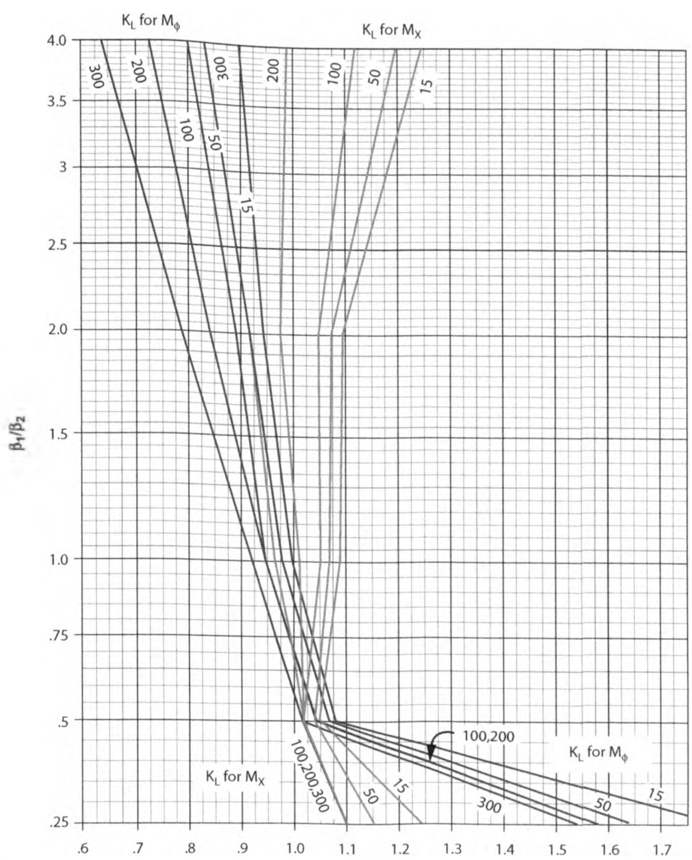

Figure 7-18. Graph of coefficients K_L for values M_ϕ & M_X from Table 7-9.

β Values for Circumferential Moment

From Table 7-10 select values of C_c and K_c and compute values of β as follows:

For N_x and N_ϕ, $\beta = \sqrt[3]{\beta_1^2\beta_2}$

For M_ϕ, $\beta = K_c\sqrt[3]{\beta_1^2\beta_2}$

For M_x, $\beta = K_c\sqrt[3]{\beta_1^2\beta_2}$

	C_c	K_c	β
N_ϕ			
N_x			
M_ϕ			
M_x			

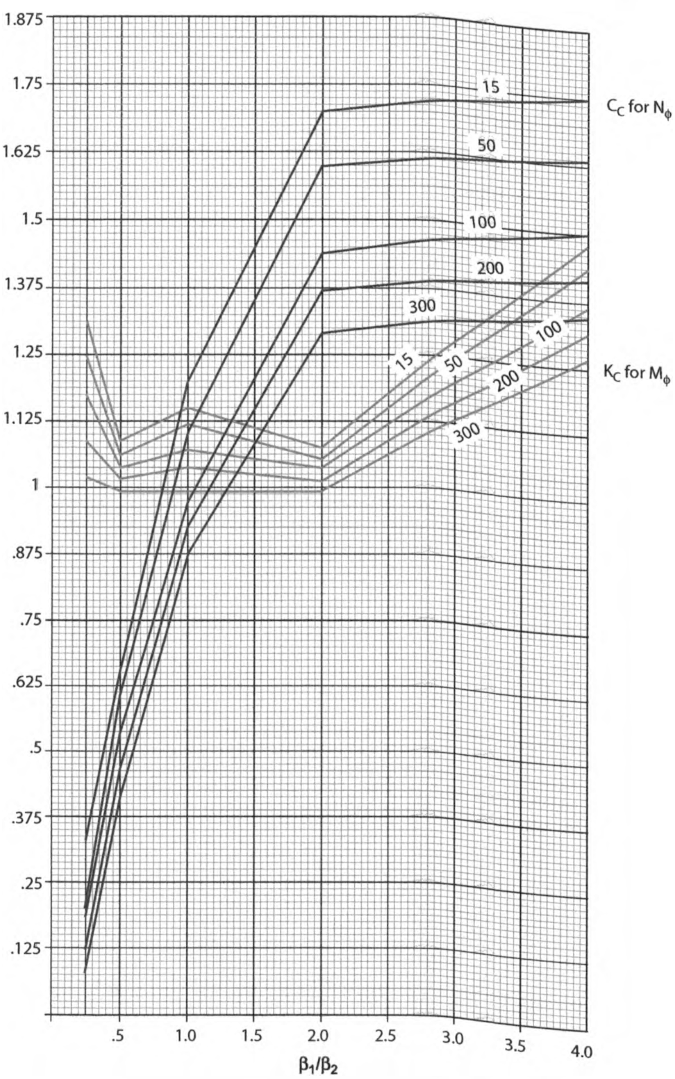

Figure 7-19. Graph of coefficients K_c & C_c for values N_ϕ & M_ϕ from Table 7-10.

Table 7-10
Coefficients for circumferential moment, M_c

β_1/β_2	γ	C_c for N_ϕ	C_c for N_x	K_c for M_ϕ	K_c for M_x
	15	0.31	0.49	1.31	1.84
	50	0.21	0.46	1.24	1.62
	100	0.15	0.44	1.16	1.45
0.25	200	0.12	0.45	1.09	1.31
	300	0.09	0.46	1.02	1.17
	15	0.64	0.75	1.09	1.36
	50	0.57	0.75	1.08	1.31
0.5	100	0.51	0.76	1.04	1.26
	200	0.45	0.76	1.02	1.20
	300	0.39	0.77	0.99	1.13
	15	1.17	1.08	1.15	1.17
	50	1.09	1.03	1.12	1.14
1	100	0.97	0.94	1.07	1.10
	200	0.91	0.91	1.04	1.06
	300	0.85	0.89	0.99	1.02
	15	1.70	1.30	1.20	0.97
	50	1.59	1.23	1.16	0.96
2	100	1.43	1.12	1.10	0.95
	200	1.37	1.06	1.05	0.93
	300	1.30	1.00	1.00	0.90
	15	1.75	1.31	1.47	1.08
	50	1.64	1.11	1.43	1.07
4	100	1.49	0.81	1.38	1.06
	200	1.42	0.78	1.33	1.02
	300	1.36	0.74	1.27	0.98

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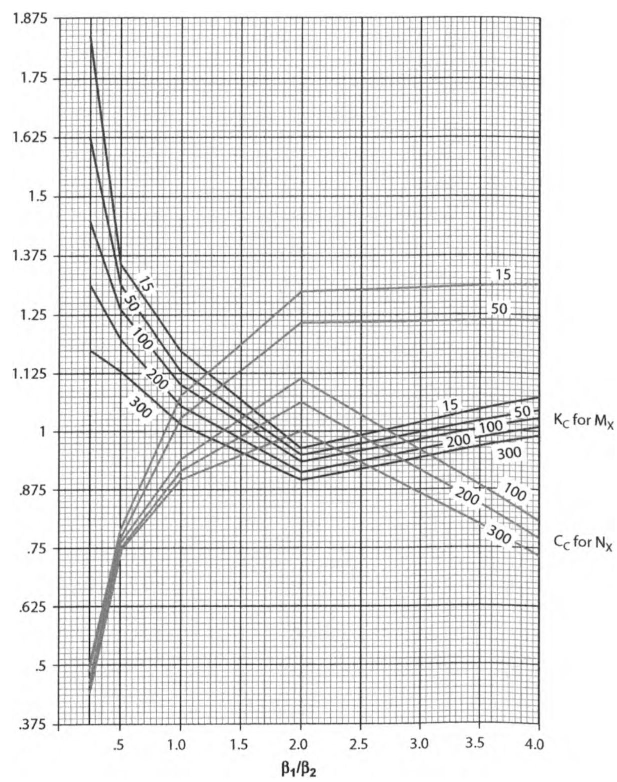


Figure 7-20. Graph of coefficients K_c & C_c for values N_x & M_x from Table 7-10.

Table 7-11
Computing stresses

	Figure	β	Value from Figure	Forces and Moments	Stress
Radial Load					
Membrane	7-21A		$\frac{N_\phi R_m}{P_r} = (\ )$	$N_\phi = \frac{(\ )P_r}{R_m}$	$\sigma_\phi = \frac{K_n N_\phi}{t}$
	7-21B		$\frac{N_x R_m}{P_r} = (\ )$	$N_x = \frac{(\ )P_r}{R_m}$	$\sigma_x = \frac{K_n N_x}{t}$
Bending	7-22A		$\frac{M_\phi}{P_r} = (\ )$	$M_\phi = (\ )P_r$	$\sigma_\phi = \frac{6K_b M_\phi}{t^2}$
	7-22B		$\frac{M_x}{P_r} = (\ )$	$M_x = (\ )P_r$	$\sigma_x = \frac{6K_b M_x}{t^2}$
Longitudinal Moment					
Membrane	7-23A		$\frac{N_\phi R_m^2 \beta}{M_L} = (\ )$	$N_\phi = \frac{(\ )C_L M_L}{R_m^2 \beta}$	$\sigma_\phi = \frac{K_n N_\phi}{t}$
	7-23B		$\frac{N_x R_m^2 \beta}{M_L} = (\ )$	$N_x = \frac{(\ )C_L M_L}{R_m^2 \beta}$	$\sigma_x = \frac{K_n N_x}{t}$
Bending	7-24A		$\frac{M_\phi R_m \beta}{M_L} = (\ )$	$M_\phi = \frac{(\ )M_L}{R_m \beta}$	$\sigma_\phi = \frac{6K_b M_\phi}{t^2}$
	7-24B		$\frac{M_x R_m \beta}{M_L} = (\ )$	$M_x = \frac{(\ )M_L}{R_m \beta}$	$\sigma_x = \frac{6K_b M_x}{t^2}$
Circumferential Moment					
Membrane	7-25A		$\frac{N_\phi R_m^2 \beta}{M_c} = (\ )$	$N_\phi = \frac{(\ )C_c M_c}{R_m^2 \beta}$	$\sigma_\phi = \frac{K_n N_\phi}{t}$
	7-25B		$\frac{N_x R_m^2 \beta}{M_C} = (\ )$	$N_x = \frac{(\ )C_c M_c}{R_m^2 \beta}$	$\sigma_x = \frac{K_n N_x}{t}$
Bending	7-26A		$\frac{M_\phi R_m \beta}{M_c} = (\ )$	$M_\phi = \frac{(\ )M_c}{R_m \beta}$	$\sigma_\phi = \frac{6K_b M_\phi}{t^2}$
	7-26B		$\frac{M_x R_m \beta}{M_c} = (\ )$	$M_x = \frac{(\ )M_c}{R_m \beta}$	$\sigma_x = \frac{6K_b M_x}{t^2}$

Table 7-12
Combining stresses

Stress Due To			σ_x				σ_ϕ			
			0°	90°	180°	270°	0°	90°	180°	270°
Radial load, P_r (Sign is (+) for outward load, (−) for inward load)	Membrane	N_ϕ								
		N_x								
	Bending	M_ϕ								
		M_x								
Longitudinal moment, M_L	Membrane	N_ϕ					+		−	
		N_x	+		−					
	Bending	M_ϕ					+		−	
		M_x	+		−					
Circumferential moment, M_c	Membrane	N_ϕ						+		−
		N_x		+		−				
	Bending	M_ϕ						+		−
		M_x		+		−				
Internal pressure, P	$\sigma_\phi = \frac{PR_m}{t} =$						+	+	+	+
	$\sigma_x = \frac{PR_m}{2t} =$		+	+	+	+				
Total, Σ										

Note: *Only absolute value of quantities are used. Combine stresses utilizing sign convention of table.*

Shear Stresses

- *Stress due to shear loads, V_L or V_c.*
  Round attachments:

$$\tau_s = \frac{V_L}{\pi r_o t}$$

$$\tau_s = \frac{V_C}{\pi r_o t}$$

Square attachments:

$$\tau_s = \frac{V_L}{4Ct}$$

$$\tau_s = \frac{V_C}{4Ct}$$

Rectangular attachments:

$$\tau_s = \frac{V_L}{4C_1 t}$$

$$\tau_s = \frac{V_C}{4C_2 t}$$

- *Stress due to torsional moment, M_T.*
  Round attachments only!

$$\tau_T = \frac{M_T}{2\pi r_o^2 t}$$

Notes

1. Figure 7-15 should be used if the vessel is in brittle (low temperature) or fatigue service. For brittle fracture the maximum tensile stress is governing. The stress concentration factor is applied to the stresses which are perpendicular to the change in section.
2. Subscripts θ and C indicate circumferential direction, X and L indicate longitudinal direction.
3. Only rectangular shapes where C_1/C_2 is between 1/4 and 4 can be computed by this procedure. The charts and graphs are not valid for lesser or greater ratios.
4. Methods of reducing stresses from local loads:
   a. Add reinforcing pad.
   b. Increase shell thickness.
   c. Add partial ring stiffener.
   d. Add circumferential ring stiffener(s).
   e. Kneebrace to reduce moment loads.
   f. Increase attachment size.
5. See Procedure 7-3 to convert irregular attachment shapes into suitable shapes for design procedure.
6. For radial loads the stress on the circumferential axis will always govern.
7. The maximum stress due to a circumferential moment is 2–5 times larger than the stress due to a longitudinal moment of the same magnitude.
8. The maximum stress from a longitudinal moment is not located on the longitudinal axis of the vessel and may be 60°–70° off the longitudinal axis. The reason for the high stresses on or adjacent to the circumferential axis is that, on thin shells, the longitudinal axis is relatively flexible and free to deform and that the loads are thereby transferred toward the circumferential axis which is less free to deform. Figures 7-23 and 7-24 do not show maximum stresses since their location is unknown. Instead the stress on the longitudinal axis is given.
9. *For attachments with reinforcing pads:* This applies only to attachments that are welded to a reinforcing plate that is subsequently welded to the vessel shell. Attachments that are welded through the pad (like nozzles) can be considered as integral with the shell.

Moment loadings for nonintegral attachments must be converted into radial loads. This will more closely approximate the manner in which the loads are distributed in shell and plate. Stresses should be checked at the edge of attachment *and* edge of reinforcing plate. The maximum height of reinforcing pad to be considered is given by:
For radial load:

$$2d_2 \max = \frac{2C_2 d_1}{C_1}$$

For longitudinal moment:

$$2d_{21} \max = \frac{4C_2 d_1}{3C_1}$$

For circumferential moment:

$$2d_{11} \max = \frac{4C_1 d_2}{3C_2}$$

Moments can be converted as follows:

$$P_r = \frac{3M_L}{4C_2}$$

or

$$P_r = \frac{3M_c}{4C_1}$$

10. This procedure is based on the principle of "flexible load surfaces." Attachments larger than one-half the vessel diameter (β>0.5) cannot be determined by this procedure. For attachments which exceed these parameters see Procedure 7-1.

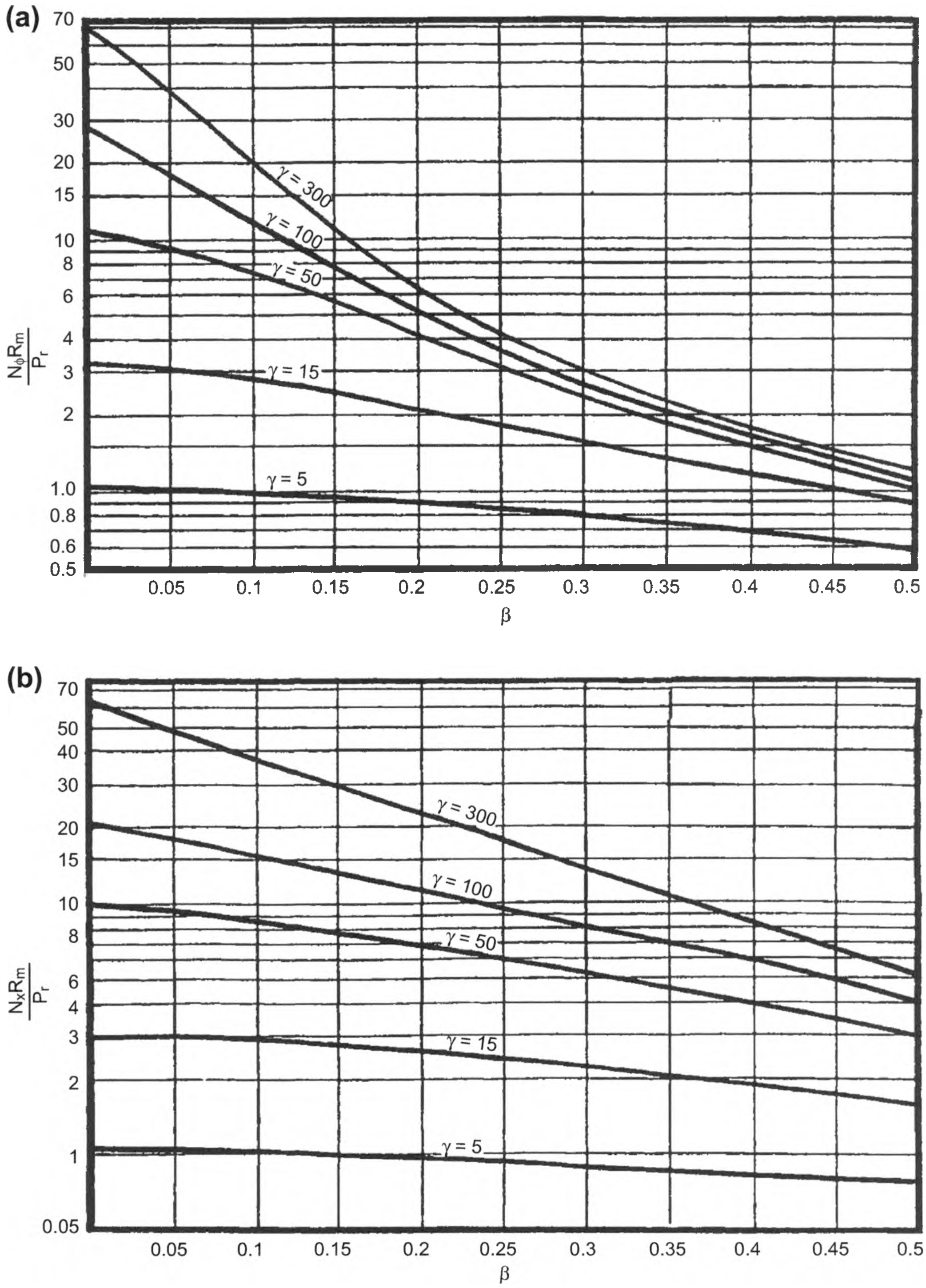


Figure 7-21. Membrane force in a cylinder due to radial load on an external attachment. (*Reprinted by permission from the Welding Research Council.*)

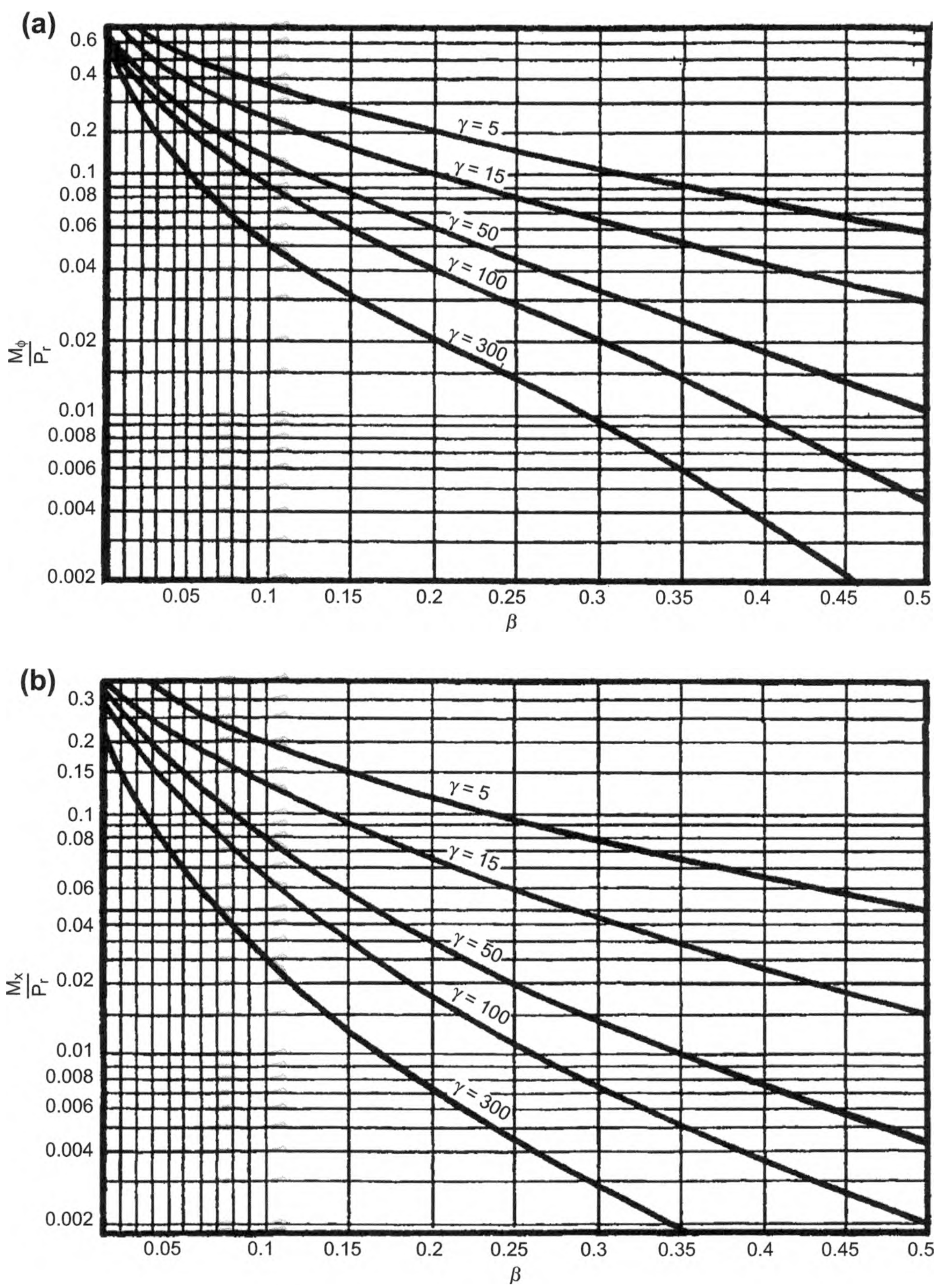


Figure 7-22. Bending moment in a cylinder due to radial load on an external attachment. (*Reprinted by permission from the Welding Research Council.*)

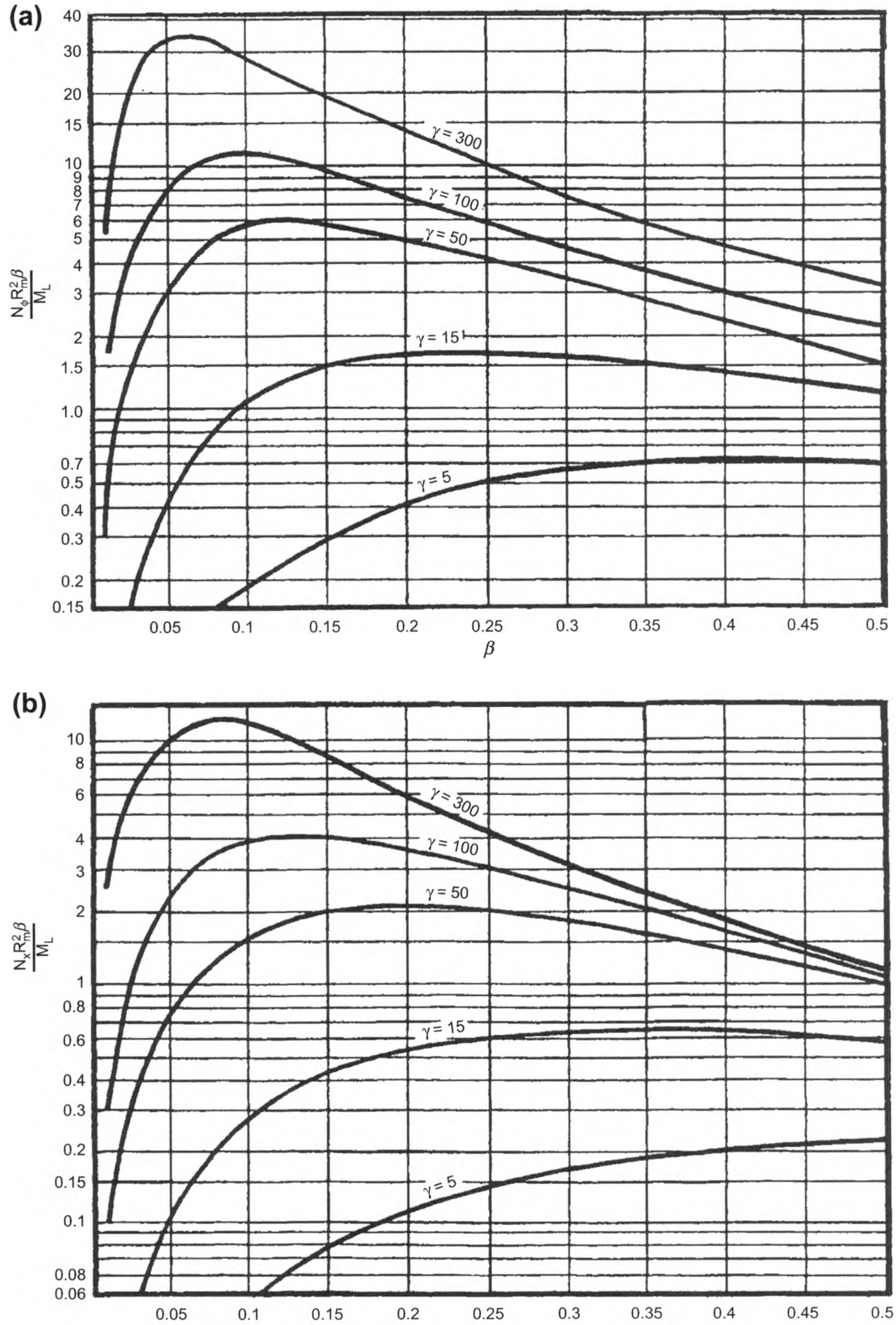


Figure 7-23. Membrane force in a cylinder due to longitudinal moment on an external attachment. (*Reprinted by permission from the Welding Research Council.*)

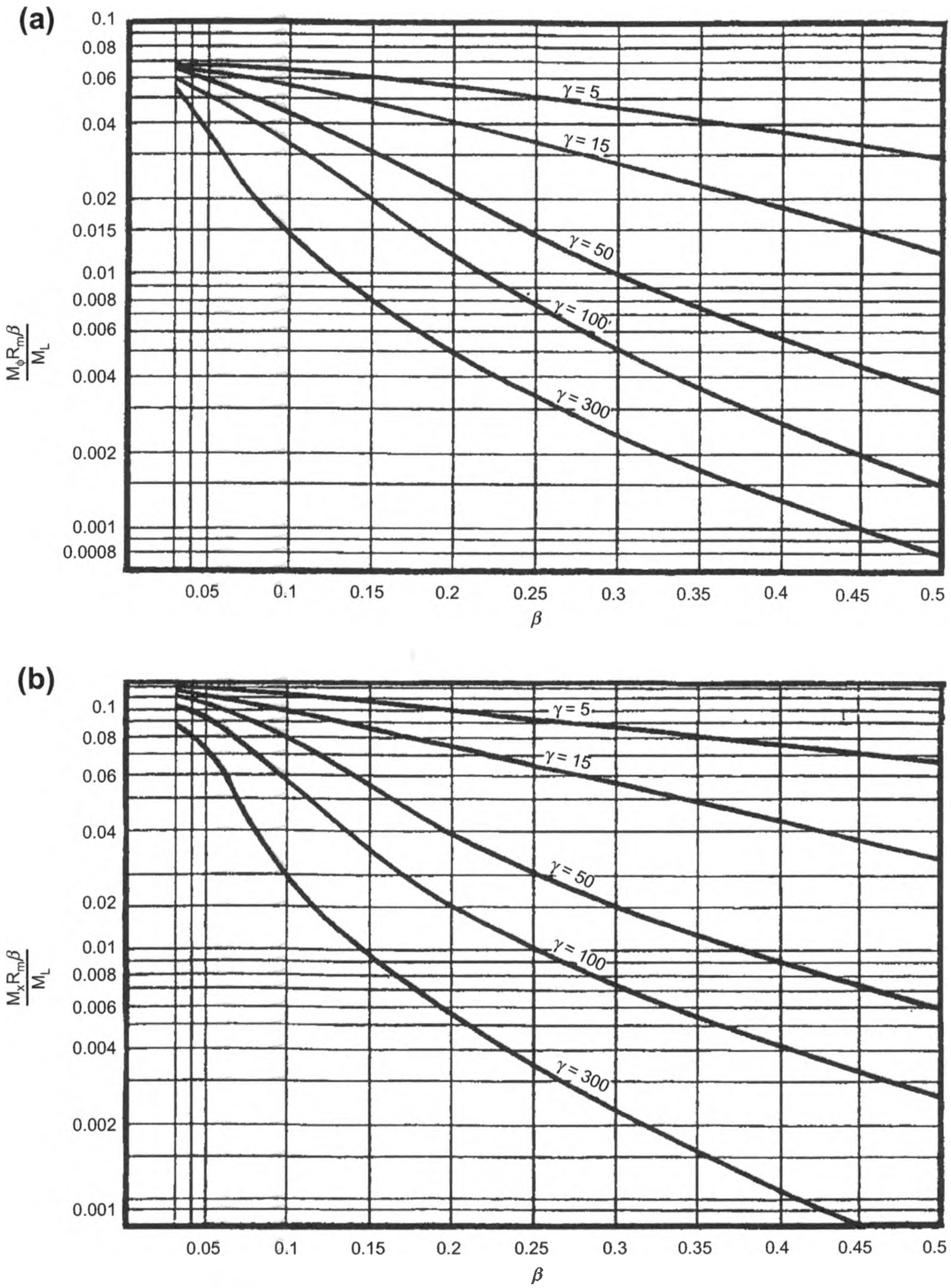


Figure 7-24. Bending moment in a cylinder due to longitudinal moment on an external attachment. (*Reprinted by permission from the Welding Research Council.*)

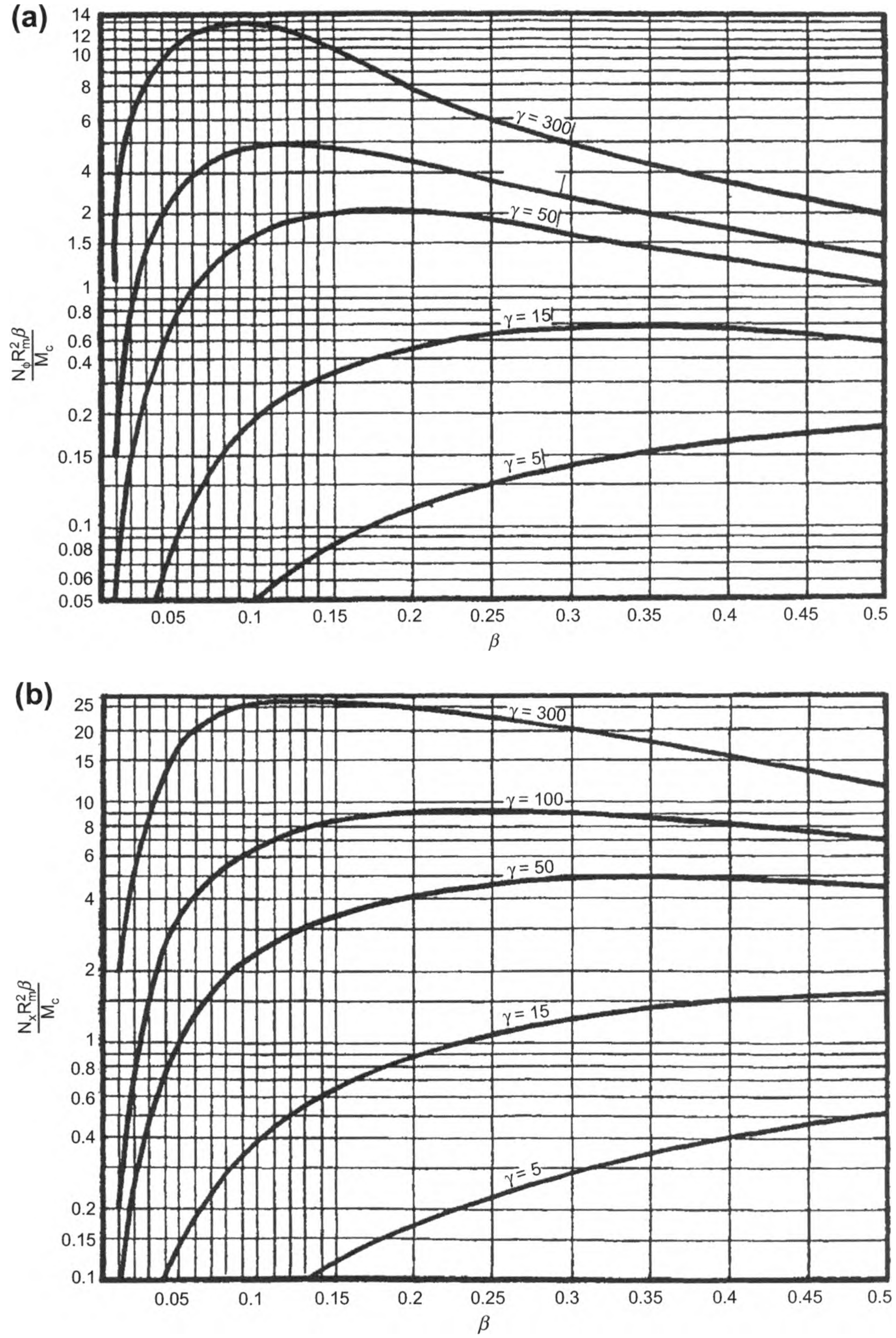


Figure 7-25. Membrane force in a cylinder due to circumferential moment on an external attachment. (*Reprinted by permission from the Welding Research Council.*)

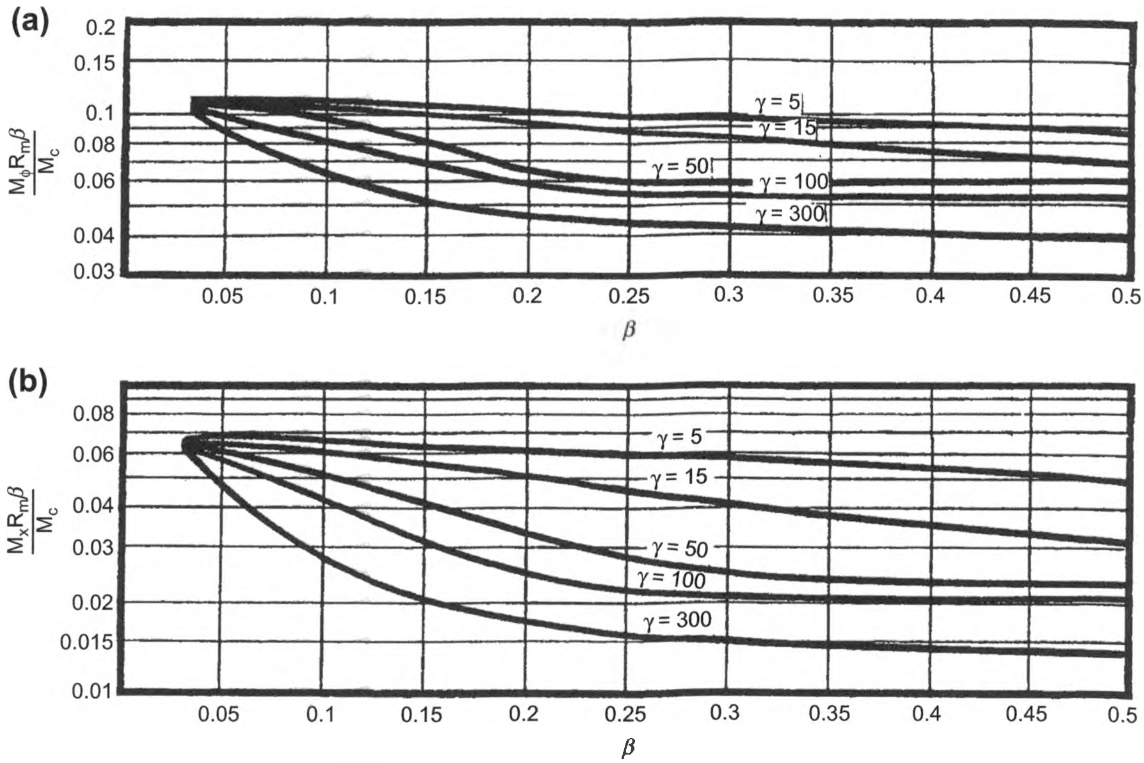


Figure 7-26. Bending moment in a cylinder due to circumferential moment on an external attachment. (*Reprinted by permission from the Welding Research Council.*)

Maximum Allowable Nozzle Loads

This procedure is an alternative work process for developing and analyzing nozzle loads on pressure vessels. It establishes the minimum criteria for design by providing the maximum allowable nozzle load by size and class of flange rating. This is an alternative work process to determining each individual nozzle load after the piping system is designed. This procedure eliminates much of the late design changes that occur when late data on nozzle loads impact either the design of the piping system or the stresses in the vessel shell.

The allowable loads and moments listed in Table 7-14 do not represent any actual loading or a real maximum allowable load or moment. Rather they are "arbitrary" maximum allowable nozzle loads. This procedure does not take into account any of the vessel parameters such as diameter, thickness, material, temperature, allowable stress, internal pressure, etc. Since the basis of Table 7-14 is the "flange rating", the associated nozzle loads are generic only.

There are two reasons for the implementation of this procedure as follows;

1. To provide the vessel fabricator with nozzle loads with which to design the vessel shell or head to which the nozzle is attached, prior to design of the piping systems.
2. To provide the piping designer with guidelines for design of piping that terminates at a vessel nozzle. Therefore, as long as the piping does not exceed the loads in Table 7-14, they automatically know that the vessel shell or head is not overstressed for this condition.

If the piping department cannot design the piping in such a way as to not exceed the values in Table 7-14, then the work process must revert back to the original work process of analyzing each specific, individual nozzle for the actual loads and resultant shell stresses. This would become an iterative work process between the vessel designer and the piping designer where actual loads will dictate the ultimate design.

In the event that the nozzle loads still result in excessive shell stresses after the iterative work process is conducted, the loadings may be reduced by recalculating the piping loads utilizing the "vessel spring rate". Often times the loads determined using the vessel spring rate will be much less than that determined by the normal process of considering the vessel as a "rigid anchor".

In general the following notes apply to this procedure;

1. Each nozzle, including those designated as "spare", but with the exception of manways and instrument nozzles, shall be designed to withstand the forces and moments specified in Table 7-14. The indicated loads are to be considered to act at the shell/head to nozzle intersection and to be true normal and tangential to the shell at that point. The effect on the shell/head shall be analyzed per an acceptable local load procedure such as WRC # 107.
2. With regard to radial load (P_r), calculations shall be made first with the force acting radially outwards in conjunction with the internal pressure and then with the force acting inwards. In the second instance, the internal pressure shall not be used to oppose the compressive stresses due to the force acting radially inwards; for this load condition a null pressure condition is to be considered to exist.
3. Values in Table 7-14 were computed by the coefficients and equations given in Table 7-13. In Table 7-13, the variables shown are "D", the nominal diameter in inches and "β", the value listed against the nozzle flange rating. These variables and equations were used in the development of Table 7-14.
4. Whenever shell or head stresses exceed the allowable stress for local loadings, the vendor shall apply

Table 7-13
Coefficients used for determination of maximum allowable nozzle loads

Flange Rating Class	150	300	600	900	1500	2500
β Value	0.6	0.7	0.8	0.9	1	1.1

Equations:

Longitudinal Bending Moment (Ft- Lbs)	$M_L = \beta \times 110 \times D^2$
Circumferential Bending moment (Ft-Lbs)	$M_\varphi = \beta \times 85 \times D^2$
Resultant Bending Moment, (Ft-Lbs)	$M_R = [M_L^2 + M_\varphi^2]^{.5} = \beta \times 140 \times D^2$
Radial Load (tension or compression) (Lbs)	$P_r = \beta \times 500 \times D$

Table 7-14
Maximum allowable nozzle loads

	FLANGE RATING							
	CLASS 150 FLANGES				CLASS 300 FLANGES			
	Force, Lbs	Bending Moment, Ft-Lbs			Force, Lbs	Bending Moment, Ft-Lbs		
NPS (Inches)	Radial Load, P_r	Longitudinal, M_L	Circumferential, M_φ	Resultant, M_R	Radial Load, P_r	Longitudinal, M_L	Circumferential, M_φ	Resultant, M_R
2	600	264	204	336	700	308	238	392
3	900	594	459	756	1050	693	536	882
4	1200	1056	816	1344	1400	1232	952	1568
6	1800	2376	1836	3024	2100	2772	2142	3528
8	2400	4224	3264	5376	2800	4928	3808	6272
10	3000	6600	5100	8400	3500	7700	5950	9800
12	3600	9504	7344	12096	4200	11088	8568	14112
14	4200	12936	9996	16464	4900	15092	11662	19208
16	4800	16896	13056	21504	5600	19712	15232	25088
18	5400	21384	16524	27216	6300	24948	19278	31752
20	6000	26400	20400	33600	7000	30800	23800	39200
24	7200	38016	29376	48384	8400	44352	34272	56448

	FLANGE RATING							
	CLASS 600 FLANGES				CLASS 900 FLANGES			
	Force, Lbs	Bending Moment, Ft-Lbs			Force, Lbs	Bending Moment, Ft-Lbs		
NPS (Inches)	Radial Load, P_r	Longitudinal, M_L	Circumferential, M_φ	Resultant, M_R	Radial Load, P_r	Longitudinal, M_L	Circumferential, M_φ	Resultant, M_R
2	800	352	272	448	900	396	306	504
3	1200	792	612	1008	1350	891	689	1134
4	1600	1408	1088	1792	1800	1584	1224	2016
6	2400	3168	2448	4032	2700	3564	2754	4536
8	3200	5632	4352	7168	3600	6336	4896	8064
10	4000	8800	6800	11200	4500	9900	7650	12600
12	4800	12672	9792	16128	5400	14256	11016	18144
14	5600	17248	13328	21952	6300	19404	14994	24696
16	6400	22528	17408	28672	7200	25344	19584	32256
18	7200	28512	22032	36288	8100	32076	24786	40824
20	8000	35200	27200	44800	9000	39600	30600	50400
24	9600	50688	39168	64512	10800	57024	44064	72576

	FLANGE RATING							
	CLASS 1500 FLANGES				CLASS 2500 FLANGES			
	Force, Lbs	Bending Moment, Ft-Lbs			Force, Lbs	Bending Moment, Ft-Lbs		
NPS (Inches)	Radial Load, P_r	Longitudinal, M_L	Circumferential, M_φ	Resultant, M_R	Radial Load, P_r	Longitudinal, M_L	Circumferential, M_φ	Resultant, M_R
2	1000	440	340	560	1100	484	374	616
3	15000	990	765	1260	1650	1089	842	1386
4	2000	1760	1360	2240	2200	1936	1496	2464
6	3000	3960	3060	5040	3300	4356	3366	5544
8	4000	7040	5440	8960	4400	7744	5984	9856
10	5000	11000	8500	14000	5500	12100	9350	15400
12	6000	15840	12240	20160	6600	17424	13464	22176
14	7000	21560	16660	27440	7700	23716	18326	30184
16	8000	28160	21760	35840	8800	30976	23936	39424
18	9000	35640	27540	45360	9900	39204	30294	49896
20	10000	44000	34000	56000	11000	48400	37400	61600
24	12000	63360	48960	80640	13200	69696	53856	88704

adequate reinforcement and or increase thickness of shell and /or nozzle locally.

5. For nozzles on a formed head or sphere, the resultant bending moment is to be compared with M_R.
6. Shear and torsion effects are omitted from consideration because they have negligible effects on final stress resultants.
7. The loadings computed from these equations shall be considered as caused by 67% thermal and 33% dead weight load.
8. Under vacuum conditions, the deflection. ς, adjacent to the nozzle should be limited to the following;

$\varsigma < .0025$ R where R is the radius of the shell or head.

Procedure 7-5: Stresses in Spherical Shells from External Local Loads [11–13]

Notation

P_r = external radial load, lb
M = external moment, in.-lb
R_m = mean radius of sphere, crown radius of F & D, dished or ellipsoidal head, in.
r_o = outside radius of cylindrical attachment, in.
C = half side of square attachment, in.
N_x = membrane force in shell, meridional, lb/in.
N_ϕ = membrane force in shell, latitudinal, lb/in.
M_x = internal bending moment, meridional, in.-lb/in.
M_ϕ = internal bending moment, latitudinal, in.-lb/in.
K_n,K_b = stress concentration factors (See Note 3)
U,S = coefficients
σ_x = meridional stress, psi
σ_ϕ = latitudinal stress, psi
T_e = thickness of reinforcing pad, in.
τ = shear stress, psi
M_T = torsional moment, in.-lb
V = shear load, lb

Procedure

To calculate stress due to radial load (P_r), and/or moment (M), on a spherical shell or head:

1. Calculate value "S" to find stresses at distance x from centerline or value "U" at edge of attachment. *Note:* At edge of attachment, S = U. Normally stress there will govern.
2. From Figures 7-29 to 7-32 determine coefficients for membrane and bending forces and enter values in Table 7-15.
3. Compute stresses in Table 7-15. These stresses are entered into Table 7-16 based on the type of stress (membrane or bending) and the type of load that produced that stress (radial load or moment).
4. Stresses in Table 7-16 are added vertically to total at bottom.

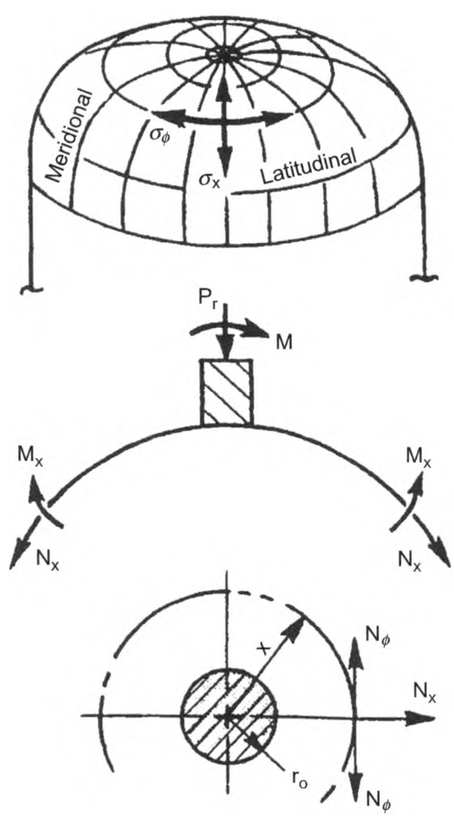


Figure 7-27. Loadings and forces at local attachments in spherical shells.

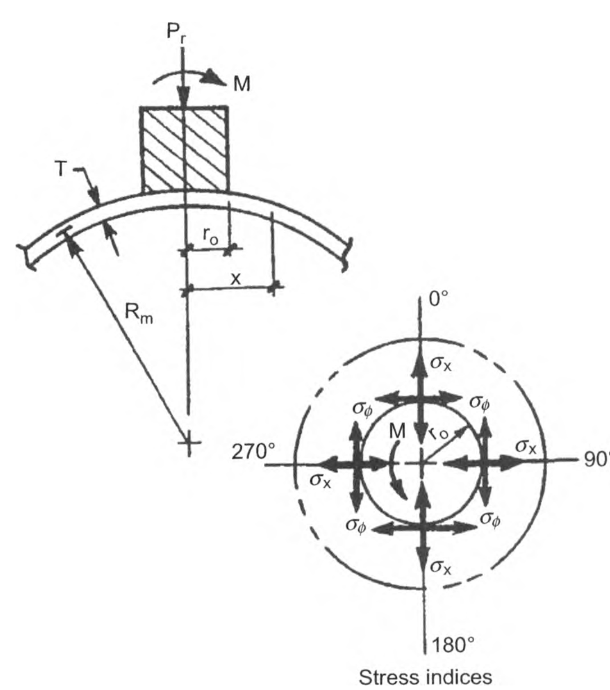


Figure 7-28. Dimensions and stress indices of local attachments.

Table 7-15
Computing stresses

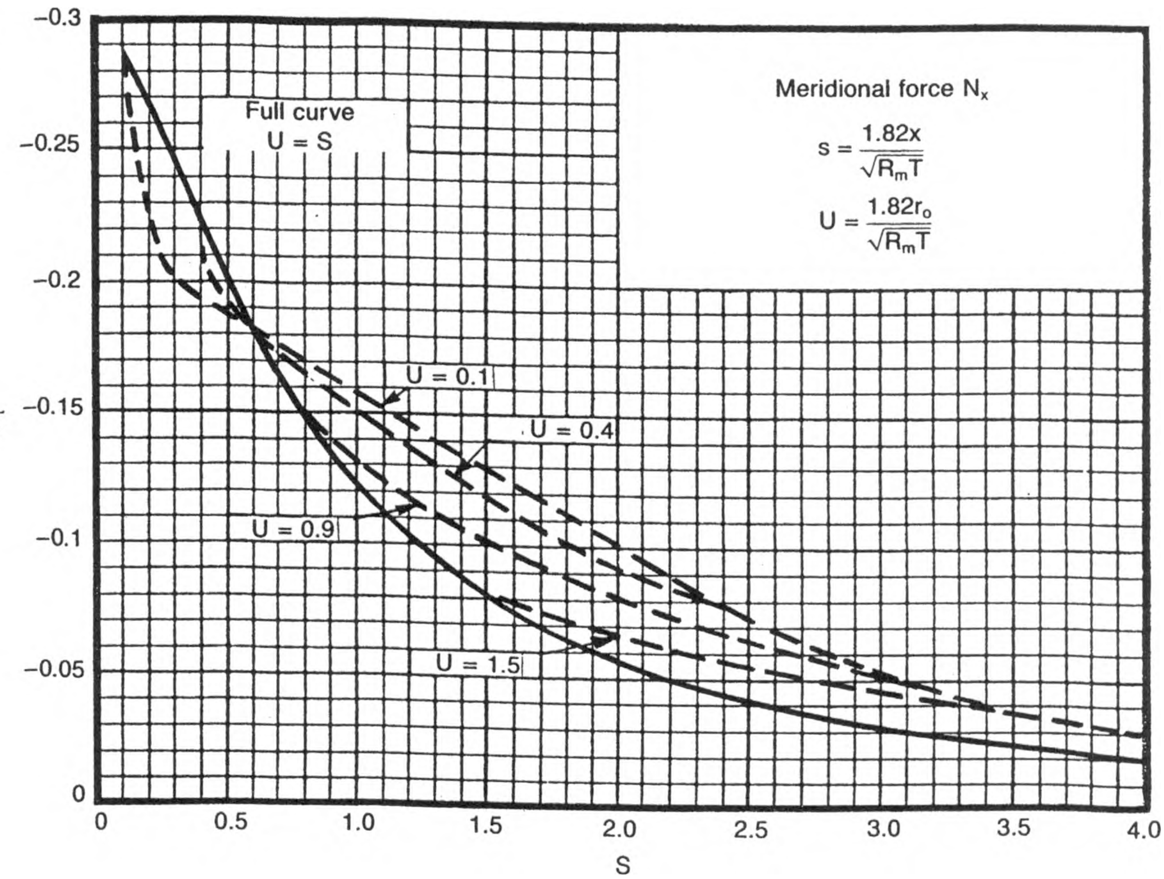
	Figure	Value from Figure	Stresses	
			Radial Load	
Membrane	7-29A	$\frac{N_x T}{P_r} = (\ )$		$\sigma_x = (\ )\frac{K_n P_r}{T^2}$
	7-29B	$\frac{N_\phi T}{P_r} = (\ )$		$\sigma_\phi = (\ )\frac{K_n P_r}{T^2}$
Bending	7-30A	$\frac{M_x}{P_r} = (\ )$		$\sigma_x = (\ )\frac{6K_b P_r}{T^2}$
	7-30B	$\frac{M_\phi}{P_r} = (\ )$		$\sigma_\phi = (\ )\frac{6K_b P_r}{T^2}$
			Moment	
Membrane	7-31A	$\frac{N_x T\sqrt{R_m T}}{M} = (\ )$		$\sigma_x = (\ )\frac{K_n M}{T^2\sqrt{R_m T}}$
	7-31B	$\frac{N_\phi T\sqrt{R_m T}}{M} = (\ )$		$\sigma_\phi = (\ )\frac{K_n M}{T^2\sqrt{R_m T}}$
Bending	7-32A	$\frac{M_x\sqrt{R_m T}}{M} = (\ )$		$\sigma_x = (\ )\frac{6K_b M}{T^2\sqrt{R_m T}}$
	7-32B	$\frac{M_\phi\sqrt{R_m T}}{M} = (\ )$		$\sigma_\phi = (\ )\frac{6K_b M}{T^2\sqrt{R_m T}}$

Table 7-16
Combining stresses

Stress Due To			σ_x 0°	90°	180°	270°	σ_ϕ 0°	90°	180°	270°
Radial load. P_t (Sign is (+) for outward radial load. (−) for inward load)	Membrane	N_x								
		N_ϕ								
	Bending	M_x								
		M_ϕ								
Moment, M	Membrane	N_x	+		−					
		N_ϕ					+		−	
	Bending	M_x	+		−					
		M_ϕ					+		−	
Total		Σ								

Note: *Only absolute values of quantities are used. Combine stresses utilizing sign convention of table.*

(a)



(b)

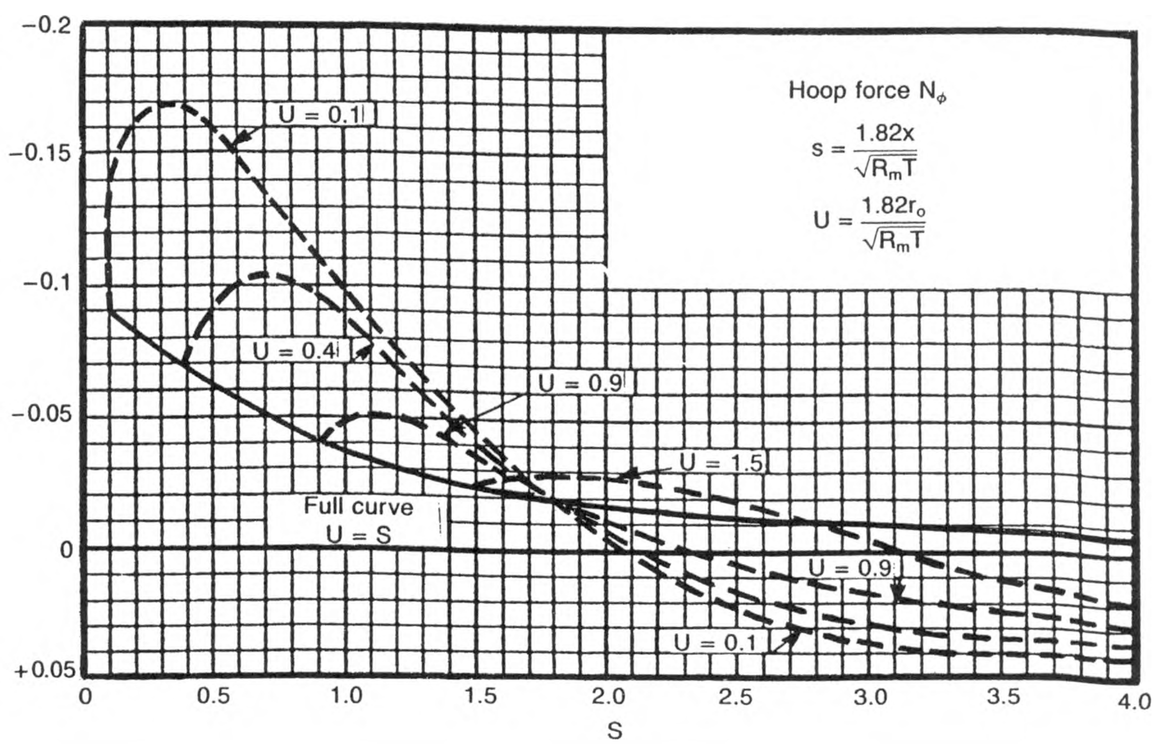
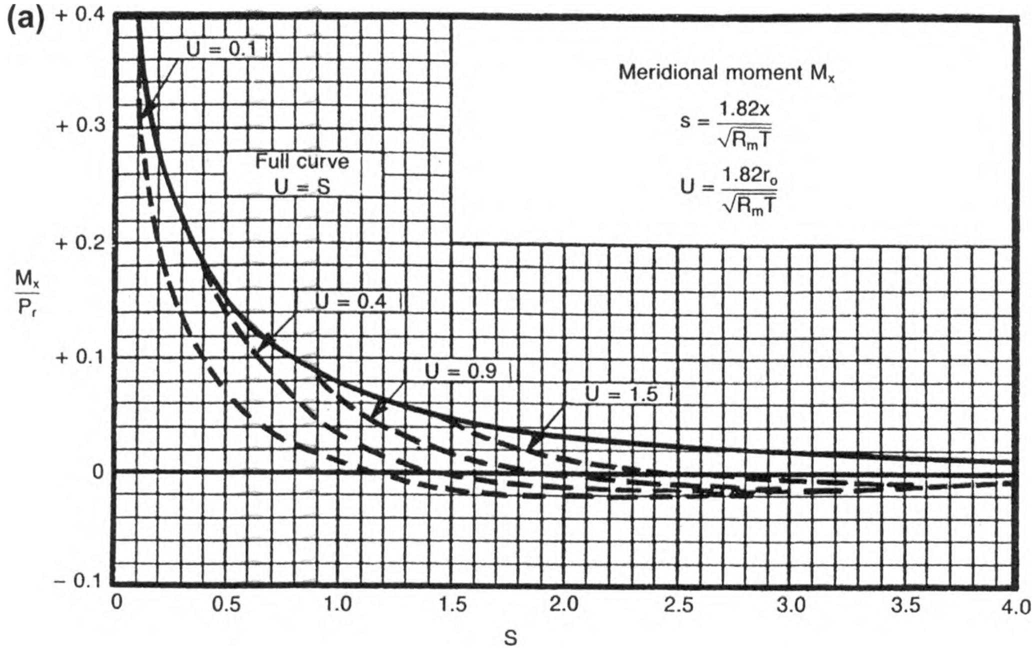


Figure 7-29. Membrane force due to P_r. (*Extracts from BS 5500:1985 are reproduced by permission of British Standards Institution, 2 Park Street, London, W1A 2BS, England. Complete copies can be obtained from national standards bodies.*)



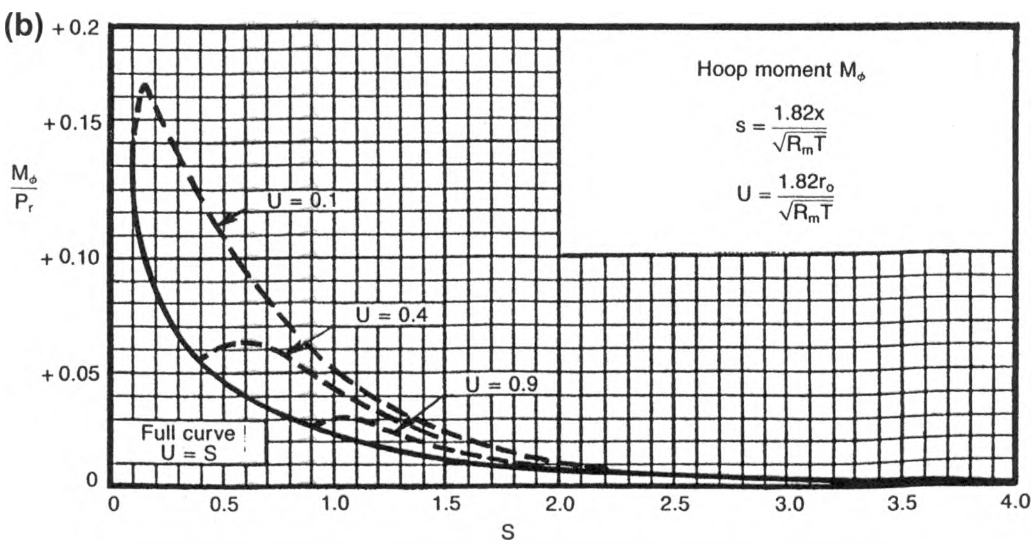


Figure 7-30. Bending moment due to P_r. (*Extracts from BS 5500:1985 are reproduced by permission of the British Standards Institution, 2 Park Street, London, W1A 2BS, England. Complete copies can be obtained from national standards bodies.*)

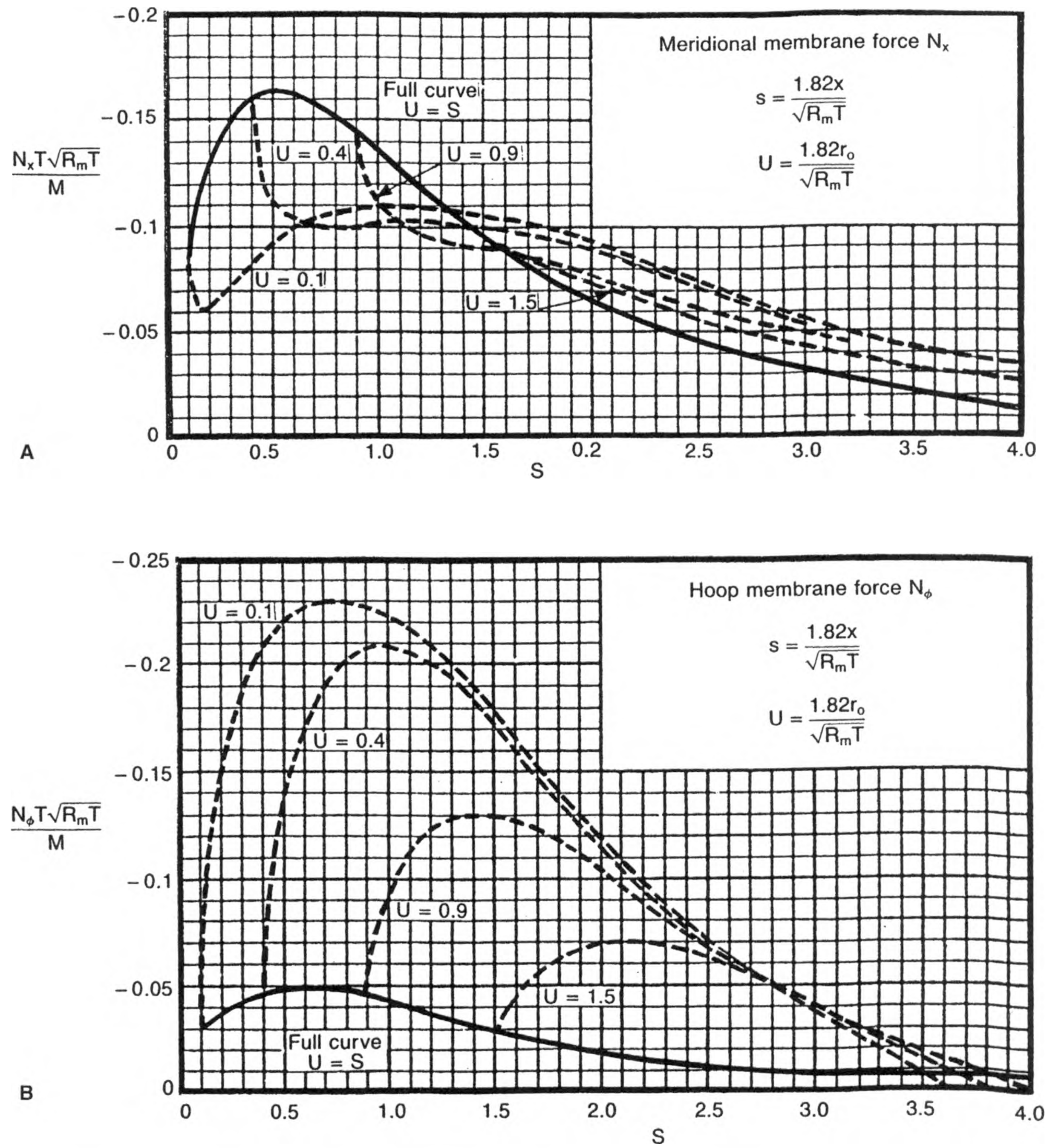


Figure 7-31. Membrane force due to M. (*Extracts from BS 5500:1985 are reproduced by permission of the British Standards Institution, 2 Park Street, London, W1A 2BS, England. Complete copies can be obtained from national standards bodies.*)

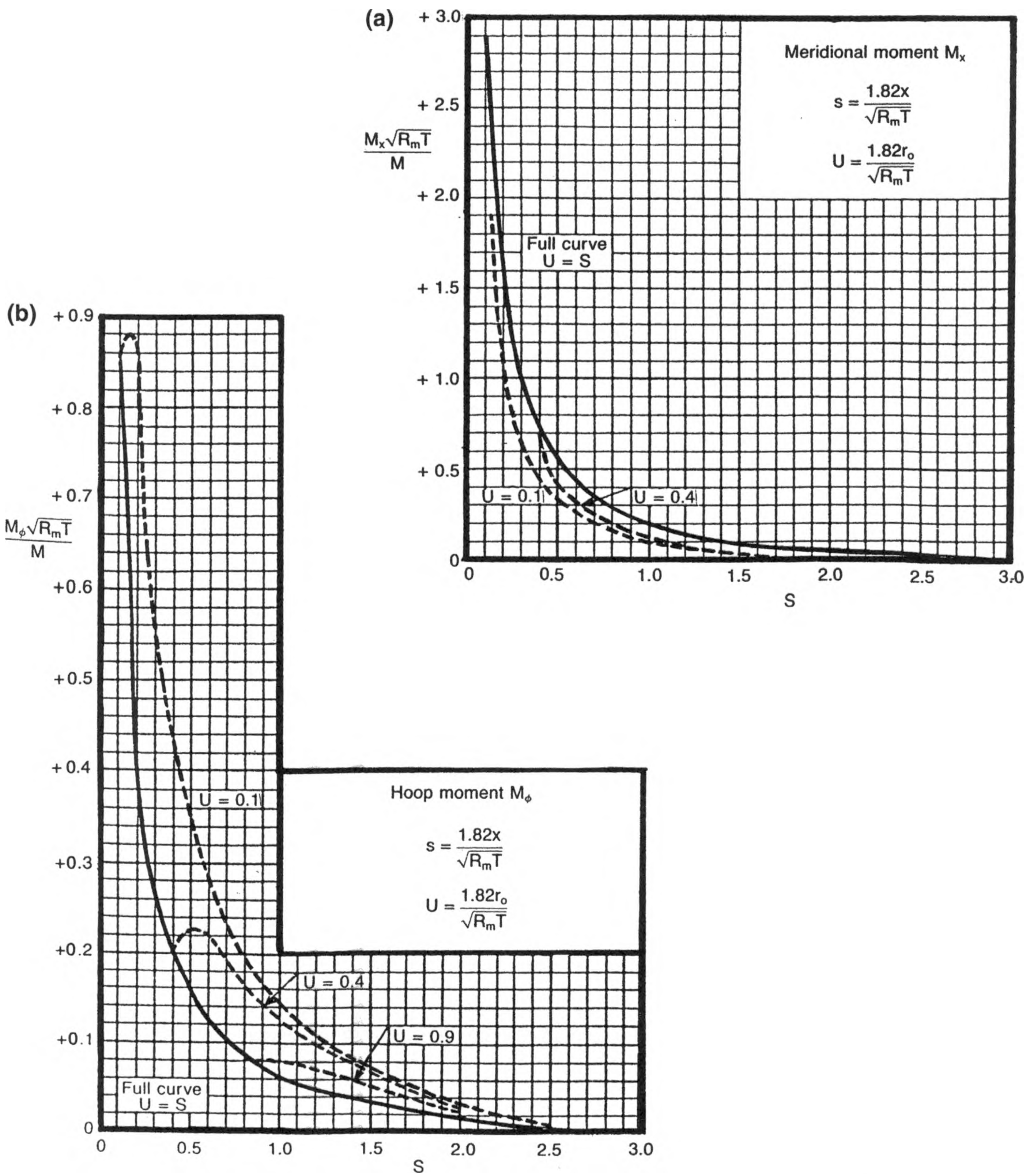


Figure 7-32. Bending moment due to M. (*Extracts from BS 5500:1985 are reproduced by permission of the British Standards Institution, 2 Park Street, London, W1A 2BS, England. Complete copies can be obtained from national standards bodies.*)

Formulas

- *For square attachments.*

$$r_o = C$$

- *For rectangular attachments.*

$$r_o = \sqrt{C_x C_\phi}$$

- *For multiple moments.*

$$M = \sqrt{M_1^2 + M_2^2}$$

- *For multiple shear forces.*

$$V = \sqrt{V_1^2 + V_2^2}$$

- *General stress equation.*

$$\sigma = \frac{N_i}{T} \pm \frac{6M_i}{T^2}$$

- *For attachments with reinforcing pads.*

$$T \text{ at edge of attachment} = \sqrt{T^2 + T_e^2}$$

$$T \text{ at edge of pad} = T$$

- *Shear stresses.*
  Due to shear load

$$\tau = \frac{V}{\pi r_o T}$$

Due to torsional moment, M_T

$$\tau = \frac{M_T}{2\pi r_o^2 T}$$

Stress Indices, Loads, and Geometric Parameters

$r_o =$

$R_m =$

$T =$

$K_n =$

$K_b =$

$P_r =$

$M =$

$$S = \frac{1.82x}{\sqrt{R_m T}}$$

$$U = \frac{1.82r_o}{\sqrt{R_m T}}$$

Notes

1. This procedure is based on the "Theory of Shallow Spherical Shells".
2. Because stresses are local and die out rapidly with increasing distance from point of application, this procedure can be applied to the spherical portion of the vessel heads as well as to complete spheres.
3. For "Stress Concentration Factors" see "Stresses in Cylindrical Shells from External Local Loads", Procedure 7-4.
4. For convenience, the loads are considered as acting on a rigid cylindrical attachment of radius r_o. This will yield approximate results for hollow attachments. For more accurate results for hollow attachments, consult WRC Bulletin 107 [11].
5. The stresses found from these charts will be reduced by the effect of internal pressure, but this reduction is small and can usually be neglected in practice. Bijlaard found that for a spherical shell with R_m/T = 100, and internal pressure causing membrane stress of 13,000 psi, the maximum deflection was decreased by only 4%–5% and bending moment by 2%. In a cylinder with the same R_m/T ratio, these reductions were about 10 times greater. This small reduction for spherical shells is caused by the smaller and more localized curvatures caused by local loading of spherical shells.

References

[1] Roark RJ. In: Formulas for Stress and Strain. 5th Edition. New York: McGraw-Hill Book Co; 1975.
[2] Isakower RI. Ring Redundants. Machine Design March 1965.
[3] Blake A. Stresses in Flanges and Support Rings. Machine Design September 1974.
[4] Samoiloff A. Investigation of Stress in Circular Rings. Petroleum Refiner July 1947.
[5] Blake A. Rings and Arcuate Beams. Product Engineering January 1963.
[6] Blodgett OW. Design of Welded Structures. The James F. Lincoln Arc Welding Foundation 1966. Section 66–4.
[7] Harvey JF. Theory and Design of Modern Pressure Vessels. 2nd Edition. Van Nostrand Reinhold Co; 1974.
[8] Roark RJ. Stresses and Deflections in Thin Shells and Curved Plates Due to Concentrated and Variously Distributed Loading. Technical Note 806, National Advisory Committee on Aeronautics; 1941.
[9] Tentative Structural Design Basis for Reactor Pressure Vessels and Directly Associated Components. PB 151987, United States Dept. of Commerce; December 1958. pp. 62–81.
[10] Dodge WG. Secondary Stress Indices for Integral Structural Attachments to Straight Pipe. Welding Research Council Bulletin No. 198; September 1974.
[11] Wichman KR, Hopper AG, Mershon JL. Local Stresses in Spherical and Cylindrical Shells Due to External Loadings. Welding Research Council Bulletin No. 107; April 1972.
[12] Bijlaard PP. Computation of the Stresses from Local Loads in Spherical Pressure Vessels or Pressure Vessel Heads. Welding Research Council Bulletin No. 34; 1957.
[13] BS 5500: Specification for Unfired Fusion Welded Pressure Vessels. London: British Standards Institute; 1985.